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Local solvability of degenerate partial differential equations. (Russian) Zbl 0716.35047

The author considers the system of partial differential equations (1) \((V \phi)(x) = g(x, \phi(x))\), where \(x \in \mathbb{R}^n\), \(\phi : \mathbb{R}^n \to \mathbb{R}^m\), \(V\) is a local smooth vector field and \(g : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^m\) is a local \(C^\infty\)-mapping.

Let \(\hat{\phi}_0\) be some formal solution of (1) and let \(\phi_0 : \mathbb{R}^n \to \mathbb{R}^m\) be a \(C^\infty\)-mapping with Taylor series in the origin of coordinates. The substitution \(\phi \to \phi + \phi_0\) leads to the equation (2) \((V \phi)(x) = \tilde{g}(x, \phi(x))\), where \(\tilde{g}(x, y) = g(x, y + \phi_0(x)) - (V \phi_0)(x)\).

If equation (2) has a \(C^\infty\)-solution \(\phi\), the equation (1) should have a \(C^\infty\)-solution \(\phi + \phi_0\). In the case of \(\phi = 0\) we can say that the formal solution of (1) is restored to the local \(C^\infty\)-solution of (1).

The author proves two theorems:

i) Let the vector field \(V\) be quasihyperbolic of order \(k\) and the Jacobi matrix \(Q(x) = (\partial g(x, y)/\partial y)|_{y=\phi_0}\) be such that \(Q(x) = O(\|x\|^k)\). Then the formal solution \(\hat{\phi}_0\) is restored to the local \(C^\infty\)-solution of (1); and

ii) Let in equation (1) the contraction of vector field \(V\) on its central manifold be quasihyperbolic of the order of \(k\) and formal solution \(\hat{\phi}_0\) be such that \(Q(x) = (\partial g(x, y)/\partial y)|_{y=\phi_0} = O(\|x\|^k)\), \(x \in L_c\). Then the formal solution of (1) is restored to the local \(C^\infty\)-solution.

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quasihyperbolic; central manifold