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In random environment the local time can be very big. (English) Zbl 0716.60087

Les processus stochastiques, Coll. Paul Lévy, Palaiseau/Fr. 1987, Astérisque 157-158, 321-339 (1988).

[For the entire collection see Zbl 0649.00017.]

Let $\mathcal{E} = \{..., E_{-2}, E_{-1}, E_0, E_1, E_2, ...\}$ be a sequence of i.i.d. random variables with $P\{E_0 < x\} = F(x)$, $0 < x < 1$, $F(0) = 0$, $F(1) = 1$. Such a sequence $\mathcal{E}$ is called a random environment; any realization of it will be denoted by the same letter $\mathcal{E}$. For any fixed sample sequence of this environment, define a random walk $\{R_n\}$ by $R_0 = 0$ and

$$P_{\mathcal{E}}\{R_{n+1} = i+1| \quad R_n = i\} = 1 - P_{\mathcal{E}}\{R_{n+1} = i-1| \quad R_n = i\} = E_i \quad (n = 0, 1, ..., \quad i = 0, \pm 1, ...).$$

Assume (i) $P\{a < E_0 < 1 - a\} = 1$ for some $0 < a <$, (ii) E log((1-E$_0$)$/E_0$) = 0 and (iii) $0 < \sigma^2 = E(\log((1 - E_0)/E_0))^2 < \infty$. Let $\xi(x, n) = \#\{k : \ 0 \le k \le n, \quad R_k = x\}, \xi(n) = \max_x \xi(x, n)$. The first result of this paper gives an upper bound for ξ (0,n) as follows: ξ (0,n)$\le \exp((1-\theta_n)\log n)$ a.s. for all but finitely many n, where $\theta_n = \exp(-C(\log_2 n)(\log_3 n)^{-1/2}\log_4 n)$, where $\log_p$ is the pth iterate of log and the meaning of a.s. is: for almost all realizations of $\mathcal{E}$ the stated inequality holds with $P_{\mathcal{E}}$-probability 1. As to the behaviour of ξ (n), it is conjectured that $\limsup_{n\to\infty} n^{-1}\xi(n) = C$ a.s. for some $0 < C = C(F) < 1$, and in the case of $P\{E_i = p\} = P\{E_i = 1 - p\} =, 0 < p <$, it is proved that $\limsup_{n\to\infty} n^{-1}\xi(n) \ge g(p)$ a.s., where $1/g(p) = 16f(x)/p + 1$, $f(x) = (2x^2 - x + 1)/(1 - x)^3$ and $x = p/(1 - p)$. These results are related to some of *P. Deheuvels* and the author [Probab. Theory Relat. Fields 72, No.2, 215-230 (1986; Zbl 0572.60070)] and the author [New perspectives in theoretical and applied statistics, Sel. Pap. 3rd Int. Meet. Stat., Bilbao/Spain 1986, 503-518 (1987; Zbl 0623.60092)], where a.s. inequalities are given for describing how small and how large ξ (0,n) can be. The paper also contains a number of interesting lemmas on various other aspects of random walks in random environments, as well as in the classical setting.

MSC:

60J55 Local time and additive functionals
60G50 Sums of independent random variables; random walks

Keywords:

random environment; random walk; random walks in random environments