Fowler, Patrick W.; Gauci, John Baptist; Goedgebeur, Jan; Pisanski, Tomaž; Sciriha, Irene
Existence of regular nut graphs for degree at most 11. (English) Zbl 1433.05153
Discuss. Math., Graph Theory 40, No. 2, 533-557 (2020).

Summary: A nut graph is a singular graph with one-dimensional kernel and corresponding eigenvector with no zero elements. The problem of determining the orders n for which d-regular nut graphs exist was recently posed by *B. Gauci* et al. ["Existence of regular nut graphs and the Fowler construction", Preprint, arXiv:1904.02229]. These orders are known for $d \leq 4$. Here we solve the problem for all remaining cases $d \leq 11$ and determine the complete lists of all d-regular nut graphs of order n for small values of d and n. The existence or non-existence of small regular nut graphs is determined by a computer search. The main tool is a construction that produces, for any d-regular nut graph of order n, another d-regular nut graph of order $n + 2d$. If we are given a sufficient number of d-regular nut graphs of consecutive orders, called seed graphs, this construction may be applied in such a way that the existence of all d-regular nut graphs of higher orders is established. For even d the orders n are indeed consecutive, while for odd d the orders n are consecutive even numbers. Furthermore, necessary conditions for combinations of order and degree for vertex-transitive nut graphs are derived.

MSC:

05C30 Enumeration in graph theory
05C50 Graphs and linear algebra (matrices, eigenvalues, etc.)
05C75 Structural characterization of families of graphs
05C90 Applications of graph theory
68R10 Graph theory (including graph drawing) in computer science

Keywords:

nut graph; core graph; regular graph; nullity

Software:

GENREG; House of Graphs; snarkhunter

Full Text: DOI

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