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**Designing multi-dimensional logistic map with fixed-point finite precision.** (English)

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Summary: In cryptographic algorithms, random sequences of longer period and higher nonlinearity are always desirable in order to increase resistance against cryptanalysis. The use of chaotic maps is an attractive choice as they exhibit properties that are suitable for cryptography. In continuous phase space of the logistic map, proper control parameters and initial state result into aperiodic trajectories. However, when the phase space of the logistic map is quantized, the trajectories terminate in finite and stable periodic orbits due to quantization error. The dynamic degradation of the logistic map can be mitigated using nonlinear feedback and cascading multiple chaotic maps. We propose a logistic map-based, finite precision multi-dimensional logistic map, that incorporates nonlinear feedback and modulus operations to perturb the chaotic trajectories. We present complexity, average cycle length and randomness analysis to evaluate the proposed method. The simulation results and analysis reveal that the proposed MDLM approach achieves longer period and higher randomness.

**MSC:**

- 68P25 Data encryption (aspects in computer science)
- 11T71 Algebraic coding theory; cryptography (number-theoretic aspects)
- 81P94 Quantum cryptography (quantum-theoretic aspects)

Cited in 1 Document

**Keywords:**

digital chaotic map; multi-dimensional logistic map; finite precision; chaotic maps; fixed-point processing

**Software:**

Diehard; Matlab

**Full Text:** [DOI](#)

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