Farber, Martin

An analogue of the Shannon capacity of a graph. (English) Zbl 0717.05070

SIAM J. Algebraic Discrete Methods 7, No. 1, 67-72 (1986).

The Shannon capacity of a graph G is the value $\alpha_s(G) = \sup_n^n \sqrt{\alpha(G^n)}$, where $\alpha(G^n)$ is the independence number of the strong product of n copies of G. The independent domination number of G, denoted $\mathcal{K}(G)$, is the smallest possible cardinality of a set which is both independent and dominating. The author introduces an analogue of the Shannon capacity, namely the $\mathcal{K}$-capacity $\mathcal{K}_s(G) = \inf_n^n \sqrt{\mathcal{K}(G^n)}$. Consider the graph $G_{\mathcal{A}} = (V, E)$ which has one vertex for each letter in an alphabet $\mathcal{A}$ and in which two vertices are adjacent if and only if the corresponding letters can be confused. The Shannon capacity of $G_{\mathcal{A}}$ yields an upper bound on the cardinality of a set of n-letter words which are pairwise nonconfusable, and the $\mathcal{K}$-capacity of $G_{\mathcal{A}}$ yields a lower bound on the size of a maximal such set. The author makes use of linear programming duality to present lower bounds on the $\mathcal{K}$- capacity and uses them to evaluate the $\mathcal{K}$-capacity of certain cycles and trees.

MSC:

05C99 Graph theory

Keywords:

Shannon capacity; independent domination number

Full Text: DOI

References:

[1] Bondy, J. A.; Murty, U. S. R., Graph theory with applications, (1976) · Zbl 1226.05083

[2] Farber, Martin, Domination and duality in weighted trees, Congr. Numer., 33, 3, (1981) · Zbl 0498.05025

[3] Farber, Martin, Independent domination in chordal graphs, Oper. Res. Lett., 1, 134, (198182) · Zbl 0495.05053 · doi:10.1016/0167-6377(82)90015-3

[4] Grötschel, M.; Lovász, L.; Schrijver, A., The ellipsoid method and its consequences in combinatorial optimization, Combinatorica, 1, 169, (1981) · Zbl 0557.10036

[5] Hell, P.; Roberts, F. S., Analogues of the Shannon capacity of a graph, Ann. Discrete Math., 12, 155, (1982) · Zbl 0501.05035

[6] Khachian, L. G., A polynomial algorithm in linear programming, Soviet Math. Dokl., 20, 191, (1979) · Zbl 0409.90079

[7] Lovász, László, On the Shannon capacity of a graph, IEEE Trans. Inform. Theory, 25, 1, (1979) · Zbl 0395.94021

[8] McEliece, RobertJ.; Posner, EdwardC., Hide and seek, data storage, and entropy, Ann. Math. Statist., 42, 1706, (1971) · Zbl 0235.05001

[9] On strong independence and the strong product of graphsin preparation

[10] Rosenfeld, M., On a problem of C. E. Shannon in graph theory, Proc. Amer. Math. Soc., 18, 315, (1967) · Zbl 0147.42801

[11] Shannon, ClaudeE., The zero error capacity of a noisy channel, Institute of Radio Engineers, Transactions on Information Theory, IT-2, 8, (1956)