Representations of relatively free profinite semigroups, irreducibility, and order primitivity.

Let $S$ be a profinite semigroup with a minimum ideal $K$, $\lambda^K: S \to T^l_K$, $\rho^K: S \to T^r_K$, representations of $S$ by left and right translations of $K$, $\Omega(K) = \{ (\lambda^K_s, \rho^K_s), s \in S \}$. The semigroup $S$ is called generalized group mapping (GGM) iff both representations $\lambda^K, \rho^K$ are faithful and weakly generalized group mapping (WGGM) iff for all distinct elements $u, v \in S$ either $\lambda^K(u) \neq \lambda^K(v)$, $\rho^K(u) \neq \rho^K(v)$ or $u, v \in K$. It is shown that if $S$ is WGGM then $S \to \Omega(K)$ is an embedding of topological semigroups and if $S$ is GGM then $\Omega(K)$ is a profinite semigroup. From here follow several join irreducibility results for pseudovarieties of semigroups, which are also join irreducible in the lattice of pseudovarieties of ordered semigroups thus are not generated by proper subpseudovarieties of ordered semigroups. As a solution of a problem by Rhodes and Steinberg a stronger form of join irreducibility for the Krohn-Rhodes complexity pseudovarieties is presented.

Reviewer: Jaak Henno (Tallinn)

MSC:
\begin{itemize}
  \item 20M05 Free semigroups, generators and relations, word problems
  \item 20M07 Varieties and pseudovarieties of semigroups
  \item 20M30 Representation of semigroups; actions of semigroups on sets
  \item 20M35 Semigroups in automata theory, linguistics, etc.
\end{itemize}

Keywords:
pseudovariety; relatively free profinite semigroup; torsion; group mapping semigroup; minimal ideal; join irreducibility; ordered semigroup

References:


