

Maclagan, Diane; Rincón, Felipe

Tropical schemes, tropical cycles, and valuated matroids. (English) Zbl 07174685
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To a subvariety Y of the n -dimensional algebraic torus T , it is possible to associate a polyhedral complex $\text{trop}(Y)$. This polyhedral complex is called the “tropicalization” of Y and encodes some fundamental data about the variety Y . In [Duke Math. J. 165, No. 18, 3379–3433 (2016; Zbl 1409.14100)] *J. Giansiracusa* and *N. Giansiracusa* proposed an upgrade of this idea for varieties to the more general framework of schemes.

In the paper under review, Maclagan and Rincón study the relation between these “tropical schemes”, the ideals in the semiring of tropical polynomials, and the framework of valuated matroids of *A. W. M. Dress* and *W. Wenzel* [Adv. Math. 93, No. 2, 214–250 (1992; Zbl 0754.05027)]. Fixing an ideal I and denoting by Y the tropical scheme defined by I , their Theorem 1.1 proves that any of the following three data determines the other two:

- The structure of the tropical scheme. (More precisely, the congruence $\text{trop}(I)$).
- The tropical ideal $\text{trop}(I)$.
- The collection of all valuated matroids associated to the graded components of the homogenization of I .

For a variety Y , the top-dimensional cells of $\text{trop}(Y)$ are endowed with certain weights that turn it into a tropical cycle. The second main result, Theorem 1.2, proves that the tropical cycle structure on a tropical variety is determined from any of the three listed objects above. This answers a question posed in [*J. Giansiracusa* and *N. Giansiracusa*, Duke Math. J. 165, No. 18, 3379–3433 (2016; Zbl 1409.14100)].

Reviewer: **Luis Ferroni** (Stockholm)

MSC:

- 14T20 Geometric aspects of tropical varieties
- 14T15 Combinatorial aspects of tropical varieties
- 14T10 Foundations of tropical geometry and relations with algebra
- 05B35 Combinatorial aspects of matroids and geometric lattices

Cited in 8 Documents

Keywords:

tropical scheme; valuated matroid

Software:

TropLi; Binomials.m2

Full Text: [DOI](#) [arXiv](#)

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