

Korpusov, M. O.; Yablochkin, D. K.

Potential theory for a nonlinear equation of the Benjamin-Bona-Mahoney-Burgers type.
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Summary: For the linear part of a nonlinear equation related to the well-known Benjamin-Bona-Mahoney-Burgers (BBMB) equation, a fundamental solution is constructed, which is combined with the second Green formula to obtain a third Green formula in a bounded domain. Then a third Green formula in the entire space is derived by passage to the limit in some class of functions. The properties of the potentials entering the Green formula in the entire space are examined. The Cauchy problem for a nonlinear BBMB-type equation is considered. It is proved that finding its classical solution is equivalent to solving a nonlinear integral equation derived from the third Green formula. The unique local-in-time solvability of this integral equation is proved by applying the contraction mapping principle. Next, the local-in-time classical solvability of the Cauchy problem is proved using the properties of potentials. Finally, the nonlinear capacity method is used to obtain a global-in-time a priori estimate for classical solutions of the Cauchy problem.

MSC:

[35A08](#) Fundamental solutions to PDEs

[35B45](#) A priori estimates in context of PDEs

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[Green formulas](#); [nonlinear capacity method](#)

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