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On stability of pseudo-isometries. (Russian) Zbl 0718.51009

In the space $\mathbb{R}^n$ we define: $\pi_1(x_1, ..., x_n) = (x_1, ..., x_k, 0, ..., 0)$, $\pi_2(x_1, ..., x_n) = (0, ..., 0, x_{k+1}, ..., x_n)$, $D(x) = |\pi_1(x)|^2 - |\pi_2(x)|^2$. A mapping $\phi: \mathbb{R}^n \to \mathbb{R}^n$ is called pseudo-isometry if for $x, y \in \mathbb{R}^n$: $D(\phi(x) - \phi(y)) = D(x - y)$. Let $U$ be a domain in $\mathbb{R}^n$, $f: U \to \mathbb{R}^n$ a homeomorphism, and $0 \leq \epsilon < 1$. The mapping $f$ is said to be a quasi-pseudo-isometry if for arbitrary $\epsilon' (\epsilon < \epsilon' < 1)$ and $x_0 \in U$ there exists $\delta > 0$ such that for every $x, y$ from the ball $B(x_0, \delta)$ the following equality holds: $D(f(x) - f(y)) = D(x - y) + \theta(x, y)/(x - y)^2$, where $|\theta(x, y)| < \epsilon'$. The set of all such mappings of $U$ will be denoted by $QPI(\epsilon, U)$.

The main result of the paper is the following theorem: Let $U \subset \mathbb{R}^n$ be a domain in $\mathbb{R}^n$, $0 \leq \epsilon < 1$ and $f \in QPI(\epsilon, U)$. Then there exist such constants $\alpha > 1, C > 0$, depending on $n$ only, and a pseudo-isometry $\phi$ such that if $U$ contains the ball $B(x_0, \epsilon R)$ then for $x \in B(x_0, R)$ the inequality $|\phi[f(x)] - x| < C \epsilon R$ holds.

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