Gurov, L. G.

On stability of pseudo-isometries. (Russian) Zbl 0718.51009

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In the space $\mathbb{R}^n$ we define: $\pi_1(x_1,...,x_n) = (x_1,...,x_k,0,...,0)$, $\pi_2(x_1,...,x_n) = (0,...,0,x_{k+1},...,x_n)$, $D(x) = |\pi_1(x)|^2 - |\pi_2(x)|^2$. A mapping $\phi : \mathbb{R}^n \to \mathbb{R}^n$ is called pseudo-isometry if for $x,y \in \mathbb{R}^n$: $\mathcal{D}(\phi(x)-\phi(y)) = \mathcal{D}(x-y)$. Let U be a domain in $\mathbb{R}^n$, f: $U \to \mathbb{R}^n$ a homeomorphism, and $0 \leq \epsilon < 1$. The mapping f is said to be a quasi-pseudo-isometry if for arbitrary ϵ' ($\epsilon < \epsilon' < 1$) and $x_0 \in U$ there exists $\delta > 0$ such that for every x, y from the ball $B(x_0,\delta)$ the following equality holds: $\mathcal{D}(f(x)-f(y)) = \mathcal{D}(x-y) + \theta(x,y)/(x-y)^2$, where $|\theta(x,y)| < \epsilon'$. The set of all such mappings of U will be denoted by QPI(ϵ,U).

The main result of the paper is the following theorem: Let $U \subset \mathbb{R}^n$ be a domain in $\mathbb{R}^n$, $0 \leq \epsilon < 1$ and $f \in QPI(\epsilon,U)$. Then there exist such constants $\alpha > 1$, $C > 0$, depending on n only, and a pseudo-isometry ϕ such that if U contains the ball $B(x_0,\epsilon R)$ then for $x \in B(x_0,R)$ the inequality $|\phi[f(x)] - x| < C\epsilon R$ holds.

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MSC:

51F20 Congruence and orthogonality in metric geometry

Keywords:

pseudo-isometry