Amanbek, Yerlan; Singh, Gurpreet; Pencheva, Gergina; Wheeler, Mary F.

Error indicators for incompressible Darcy flow problems using enhanced velocity mixed finite element method. (English) Zbl 1437.76019

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Summary: Local mesh adaptivity serves as a practical tool in numerical simulations to accurately capture features of interest while reducing computational time and memory requirements. In this work, we suggest a refinement strategy based on pressure and flux error estimates for numerical simulation of an incompressible, single phase flow and transport process in the subsurface porous media. We derive a posteriori error estimates for an Enhanced Velocity Mixed Finite Element Method (EVMFEM) as a spatial domain decomposition approach. We note that the flux errors play an important role in coupled flow and transport systems later demonstrated using numerical experiments. A comparison between explicit (residual based) error estimators and an implicit error estimator; based upon the post-processing proposed by *T. Arbogast* and *Z. Chen* [Math. Comput. 64, No. 211, 943–972 (1995; Zbl 0829.65127)], shows that the latter performs better. A residual-based error estimator for pressure was found to be both computationally efficient while sufficiently indicating the large error subdomains. Numerical studies are also presented that confirm our theoretical derivations while demonstrating the advantages of post-processing in detecting velocity errors.

MSC:

76M10 Finite element methods applied to problems in fluid mechanics
65N30 Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs
76S05 Flows in porous media; filtration; seepage
65N15 Error bounds for boundary value problems involving PDEs

Keywords:

a posteriori error analysis; enhanced velocity mixed finite element method; error estimates

Full Text: DOI

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