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An asymptotic numerical method for continuation of spatial equilibria of special Cosserat rods. (English) [Zbl 1440.74480](#)

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Summary: We present an efficient numerical scheme based on asymptotic numerical method for continuation of spatial equilibria of special Cosserat rods. Using quaternions to represent rotation, the equations of static equilibria of special Cosserat rods are posed as a system of thirteen first order ordinary differential equations having cubic nonlinearity. The derivatives in these equations are further discretized to yield a system of cubic polynomial equations. As asymptotic-numerical methods are typically applied to polynomial systems having quadratic nonlinearity, a modified version of this method is presented in order to apply it directly to our cubic nonlinear system. We then use our method for continuation of equilibria of the follower load problem and demonstrate our method to be highly efficient when compared to conventional solvers based on the finite element method. Finally, we demonstrate how our method can be used for computing the buckling load as well as for continuation of postbuckled equilibria of hemitropic rods.

MSC:

- [74S99](#) Numerical and other methods in solid mechanics
- [65N99](#) Numerical methods for partial differential equations, boundary value problems
- [74S05](#) Finite element methods applied to problems in solid mechanics
- [74G60](#) Bifurcation and buckling
- [74K10](#) Rods (beams, columns, shafts, arches, rings, etc.)

Keywords:

[Cosserat rods](#); [asymptotic numerical method](#); [elasticity](#); [buckling](#); [continuation](#)

Software:

[AUTO](#)

Full Text: [DOI](#)

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