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**Generalized stochastic resonance for a fractional noisy oscillator with random mass and random damping.** (English) [Zbl 1436.34061](#)

*J. Stat. Phys.* 178, No. 5, 1201-1216 (2020).

**Summary:** In this paper, we consider the random dichotomous fluctuations on both mass and damping in a fractional oscillator, which is subject to an additive fractional Gaussian noise and driven by a periodic force. In order to investigate the generalized stochastic resonance (GSR) phenomena, we acquire the exact expression of the first-order moment of system's steady response by applying the generalized fractional Shapiro-Loginov formula and Laplace transform. Additionally, we discuss the evolutions of the output amplitude amplification (OAA) with driving frequency, noise parameters, fractional order, and damping strength. It is observed that the non-monotonic resonance behaviors of one-peak GSR, double-peak GSR and triple-peak GSR exist in this fractional system. Moreover, the interplay of mass fluctuation, damping fluctuation, and memory effect can generate a rich variety of non-equilibrium cooperation phenomena, especially the stochastic multi-resonance (SMR) behaviors. It is worth emphasizing that the triple-peak GSR was not observed in previously proposed fractional oscillator subjected to dichotomous noise. Finally, the numerical simulations are also carried out based on predictor-corrector approach to verify the effectiveness of analytic result.

**MSC:**

- [34F15](#) Resonance phenomena for ordinary differential equations involving randomness
- [34C15](#) Nonlinear oscillations and coupled oscillators for ordinary differential equations
- [34A08](#) Fractional ordinary differential equations and fractional differential inclusions
- [34F05](#) Ordinary differential equations and systems with randomness
- [37C60](#) Nonautonomous smooth dynamical systems
- [34C60](#) Qualitative investigation and simulation of ordinary differential equation models

**Keywords:**

generalized stochastic resonance (GSR); fractional oscillator (FO); random mass; random damping; dichotomous noise fluctuation; stochastic multi-resonance

**Software:**

FODE

**Full Text:** [DOI](#)

**References:**

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