

**Cohn, P. M.**

**Algebra. Volume 3. 2nd ed.** (English) [Zbl 0719.00002](#)  
Chichester etc.: John Wiley & Sons. xii, 474 p. £55.00 (1991).

The author expanded (practically doubled) and reorganized the material in the second volume of his authoritative algebra textbook series London (1977; [Zbl 0341.00002](#)). The resulting material has been divided into two. This volume, under review, contains the more advanced parts from the former volume 2. The contents are as follows: Chapter 1. Universal algebra - algebras and homomorphisms, congruences and the isomorphism theorems, free algebras and varieties, abstract dependence relations, the diamond lemma, ultraproducts, the natural numbers. Chapter 2, multilinear algebra - graded algebras, free algebras and tensor algebras, the Hilbert series of a graded algebra or module, the exterior algebra on a module. Chapter 3. Homological algebra - additive and abelian categories, functors on abelian categories, the category of modules over a ring, homological dimension, derived functors, Ext and Tor and the global dimension of a ring. Chapter 4. Further group theory - group extensions, the Frattini subgroup and the Fitting subgroup, Hall subgroups, the transfer, free groups, commutators, linear groups. Chapter 5. Further field theory - algebraic dependence, simple transcendental extensions, separable and p-radical extensions, derivations, linearly disjoint extensions, infinite algebraic extensions, Galois cohomology, Kummer extensions. Chapter 6. Algebras - the Krull-Schmidt theorem, the projective cover of a module, semiperfect rings, equivalence of module categories, the Morita context, projective, injective and flat modules, Hochschild cohomology and separable algebras. Chapter 7. Central simple algebras - simple Artinian rings, the Brauer group, the reduced norm and trace, quaternion algebras, crossed products, change of base field, cyclic algebras. Chapter 8. Quadratic forms and ordered fields - the Clifford algebra of a quadratic space, the spinor norm, formally real fields, the Witt ring of a field, the symplectic group, the orthogonal group, quadratic forms in characteristic 2. Chapter 9, Noetherian rings and polynomial identities - rings of fractions, principal ideal domains, skew polynomials and Laurent series, Goldie's theorem, PI-algebras, varieties of PI-algebras and Regev's theorem, generic matrix rings and central polynomials, generalized polynomial identities. Chapter 10. Rings without finiteness assumptions - the endomorphism ring of a vector space, the density theorem revisited, primitive rings, semiprimitive rings and the Jacobson radical, algebras without a unit element, semiprime rings and nilradicals, prime PI-algebras, simple algebras with minimal right ideals, firs and semifirs. Chapter 11. Skew fields - generalities, the Dieudonné determinant, free fields, valuations on skew fields, pseudolinear extensions.

The topics, in the volume, are covered to a considerable depth. For instance, in the section on the Hilbert series of a graded module, the author proves the Hilbert-Serre theorem on the rationality of the series and the Golod-Shafarevich criterion for a finitely generated algebra to have infinite dimension; in the exercises that follow are given examples of finitely generated algebras of infinite dimensions and of finitely generated groups with finite exponent that are infinite. The text contains many worked examples and hundreds of exercises of varying difficulty. The book includes some new proofs as well as new presentations that simplify the exposition. This volume can be used as a textbook for graduate algebra courses. It is certainly suitable for self study. The writing of this book is masterful.

Reviewer: [A.A.Iskander \(Lafayette\)](#)

**MSC:**

- 00A05 Mathematics in general
- 16-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to associative rings and algebras
- 08-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to general algebraic systems
- 13-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to commutative algebra
- 14-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to algebraic geometry
- 20-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to group theory
- 18-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to category theory

Cited in **3** Reviews  
Cited in **20** Documents

**Keywords:**

algebra textbook; Universal algebra; multilinear algebra; Homological algebra; group theory; field theory; Algebras; Quadratic forms; Rings