Cristea, Mihai

A removable singularity condition for maps in R^n. (English) Zbl 0719.30012

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In this brief article, the author uses the notation $D^+f(x)$ for

$$\limsup_{y\to x} \|f(y)-f(x)\|/\|y-x\|.$$

With D a domain in $\mathbb{R}^n$, the idea is to study those f: $D\to\mathbb{R}^n$ having $D^+f(x)<\infty$ apart from points x lying in a certain exceptional set K, assumed to have $m_{n-1}(K)=0$. However, he does assume that D^+f, as restricted to $D\setminus K$, is locally bounded near points of K. Indeed, in his main theorem, where he also assumes that f is continuous, he proves without difficulty that the local bound for D^+f (on $D\setminus K$) is a local Lipschitz constant, and therefore a local bound for D^+f (on D).

The reviewer feels that the corollaries are somewhat overstated. Denoting the branch set as usual by B_f, the conclusion of Corollary 2 that the Hausdorff dimension of B_f exceeds n-2 (implicitly if $B_f\neq\emptyset$), seems to overlook the standard double-twist example expressed using cylindrical coordinates in $\mathbb{R}^3$ by (r,Θ,z)→ $(r,2\Theta,z)$. For this particular f one has $D^+f\le 2$ without exception, yet B_f is the entire z-axis. The argument associated to Corollary 2, which the above example does not refute, is basically only that $B_f=\emptyset$ if $m_{n-2}(B_f)=0$.

MSC:

30C65 Quasiconformal mappings in $\mathbb{R}^n$, other generalizations