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On the Maurey-Pisier and Dvoretzky-Rogers theorems. (English) Zbl 1450.46007

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Given $2 \leq q < \infty$, a Banach space E has cotype q whenever there is a constant $C > 0$ such that for each choice of finitely many vectors $x_1, \dots, x_n \in X$ we have

$$\Big(\sum_{k=1}^{n} \|x_k\|^q\Big)^{1/q} \leq C\Big(\int_0^1 \Big\|\sum_{k=1}^{n} r_k(t)x_k\Big\|^2\Big)^{1/2},$$

where $r_k : [0,1] \to \mathbb{R}$ denotes the k-th Rademacher function. Given $1 \leq p \leq q < \infty$, a (linear) operator $T : E \to F$ in Banach spaces is absolutely (q,p)-summing whenever there is a constant $C > 0$ such that, for each choice of finitely many vectors $x_1, \dots, x_n \in X$, we have

$$\Big(\sum_{k=1}^{n} \|x_k\|^q\Big)^{1/q} \leq C \sup_{\|x^*\|_{X^*} \leq 1} \Big(\sum_{k=1}^{n} |x^*(x_k)|^p\Big)^{1/p}.$$

The Maurey-Pisier theorem shows that for every infinite dimensional Banach space X the best $2 \leq a < \infty$ such that X has cotype a, denoted by $\cot X$, equals the best $2 \leq a < \infty$ such that the identity id_X on X is $(a,1)$-summing. Moreover, the Dovoretzky-Rogers theorem tells us that the identity id_X is not absolutely (q,p)-summing for any infinite dimensional Banach space X whenever $1/p - 1/q < 1/2$, and that this estimate is even sharp. Mainly based on the work of Maurey and Pisier, the main result here is as follows: Let X be an infinite dimensional Banach space and $1 \leq b < \infty$.

(i) If $b \geq (\cot X)^*$, the conjugate index, then

$$\inf\{a : \mathrm{id}_X \text{ is absolutely } (a,b)\text{-summing}\} = \infty.$$

(ii) If $b < (\cot X)^*$, then

$$\inf\{a : \mathrm{id}_X \text{ is absolutely } (a,b)\text{-summing}\} = \frac{b \cot X}{b + \cot X - b \cot X}.$$

Note that (i) completes the information given by the DR-theorem, and for $b = 1$ both statements together form the MP-theorem. As an application the authors prove an extension of a well-known Grothendieck-type theorem of Kwapień for operators from ℓ_1 into ℓ_p (Theorem 8) which, in fact, is part of a similar full characterization given much earlier in Proposition 5.2 of [*G. Bennett*, Duke Math. J. 44, 603–639 (1977; Zbl 0389.47015)].

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MSC:

46B07 Local theory of Banach spaces
46A32 Spaces of linear operators; topological tensor products; approximation properties
47H60 Multilinear and polynomial operators

Keywords:

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