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**Robust topology optimization for multi-material structures under interval uncertainty.** (English) [Zbl 07193096](#)

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**Summary:** In this paper, we propose an efficient method to design robust multi-material structures under interval loading uncertainty. The objective of this study is to minimize the structural compliance of linear elastic structures. First, the loading uncertainty can be decomposed into two unit forces in the horizontal and vertical directions based on the orthogonal decomposition, which separates the uncertainty into the calculation coefficients of structural compliance that are not related to the finite element analysis. In this manner, the time-consuming procedure, namely, the nested double-loop optimization, can be avoided. Second, the uncertainty problem can be transformed into an augmented deterministic problem by means of uniform sampling, which exploits the coefficients related to interval variables. Finally, an efficient sensitivity analysis method is explicitly developed. Thus, the robust topology optimization (RTO) problem considering interval uncertainty can be solved by combining orthogonal decomposition with uniform sampling (ODUS). In order to eliminate the influence of numerical units when comparing the optimal results to deterministic and RTO solutions, the relative uncertainty related to interval objective function is employed to characterize the structural robustness. Several multi-material structure optimization cases are provided to demonstrate the feasibility and efficiency of the proposed method, where the magnitude uncertainty, directional uncertainty, and combined uncertainty are investigated.

**MSC:**

74 Mechanics of deformable solids

90 Operations research, mathematical programming

**Keywords:**

multi-material structures; loading uncertainty; evolutionary structural optimization; ODUS method; robust

**Software:**

[top88.m](#); [top.m](#); [topopt\\_multi](#)

**Full Text:** [DOI](#)

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