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A generalized matrix Krylov subspace method for TV regularization. (English) Zbl 07193286
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Summary: This paper presents efficient algorithms to solve both TV/L1 and TV/L2 models of images contaminated by blur and noise. The unconstrained structure of the problems suggests that one can solve a constrained optimization problem by transforming the original unconstrained minimization problem to an equivalent constrained minimization one. An augmented Lagrangian method is developed to handle the constraints when the model is given with matrix variables, and an alternating direction method (ADM) is used to iteratively find solutions of the subproblems. The solutions of some subproblems are belonging to subspaces generated by application of successive orthogonal projections onto a class of generalized matrix Krylov subspaces of increasing dimension. We give some theoretical results and report some numerical experiments to show the effectiveness of the proposed algorithms.

MSC:

- 65 Numerical analysis
- 90 Operations research, mathematical programming

Keywords:

matrix Krylov subspace; total variation (TV); regularization; color image restoration; discrete ill-posed problems

Software:

ALGENCAN; Regularization tools

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