Prasad, Dipendra
Multiplicities under basechange: finite field case. (English) [Zbl 1447.11064]
J. Algebra 556, 1101-1114 (2020).

In this paper, the author proves a general proposition relating multiplicities for the restriction of a representation of a group to a subgroup under base change for reductive groups over finite fields, giving several interesting applications.

To explain one of the results, let $H \subset G$ be two connected algebraic groups over a finite field $F = F_q$ of order $q$, and $\pi_1, \pi_2$ be irreducible uniform (i.e., a virtual sum of Deligne-Lusztig representations $R(T, \theta)$ for varying maximal tori $T$ of $G$ and characters $\theta : T(F) \to \mathbb{C}^\times$) representations of $G(F), H(F)$ respectively, both representations assumed to have base change to the unique quadratic extension $E$ of $F$, denoted $\pi_E^1, \pi_E^2$ respectively. It is further assumed that $\pi_E^1, \pi_E^2$ are irreducible representations of $G(E), H(E)$ invariant under $\langle \sigma \rangle = \text{Gal}(E/F)$. The Shintani character identity then fixes unique extensions of $\pi_E^1, \pi_E^2$ to representations $\tilde{\pi}_E^1, \tilde{\pi}_E^2$ of $G(E) \rtimes \langle \sigma \rangle, H(E) \rtimes \langle \sigma \rangle$ respectively. One main result of the paper under review, which generalises Theorem 1 in [D. Prasad, Compos. Math. 119, No. 3, 335–345 (1999; Zbl 0969.22008)], may then be stated as

$$2m(\tilde{\pi}_E^1, \tilde{\pi}_E^2) = m(\pi_E^1, \pi_E^2) + m(\pi_1, \pi_2),$$

where $m(\pi_1, \pi_2) = \dim \text{Hom}_{H(F)}(\pi_1, \pi_2)$ etc.

The result is then used to calculate certain multiplicities for cuspidal representations of general linear groups which become principal series representations under base change for which multiplicities can be calculated by geometric methods. This is applied mainly to calculate which one-dimensional representations of $\text{GL}_n(E)$ and $\text{GL}_n(F) \times \text{GL}_n(F)$ appear in a cuspidal representation of $\text{GL}_{2n}(F)$. Several other results with interesting corollaries are proved.

Reviewer: Ramdin Mawia (Kolkata)

MSC:
11F70 Representation-theoretic methods; automorphic representations over local and global fields
22E55 Representations of Lie and linear algebraic groups over global fields and adèle rings

Keywords:
Deligne-Lusztig representations; basechange; Shintani character identity; distinguished representations; branching laws

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References:

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