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Semilattice sums of algebras and Mal’tsev products of varieties. (English) [Zbl 07203511]

Summary: The Mal’tsev product of two varieties of similar algebras is always a quasivariety. We consider
the question of when this quasivariety is a variety. The main result asserts that if \( V \) is a strongly irregular
variety with no nullary operations and at least one non-unary operation, and \( S \) is the variety, of the
same type as \( V \), equivalent to the variety of semilattices, then the Mal’tsev product \( V \circ S \) is a variety.
It consists precisely of semilattice sums of algebras in \( V \). We derive an equational base for the product
from an equational base for \( V \). However, if \( V \) is a regular variety, then the Mal’tsev product may not be
a variety. We discuss various applications of the main result, and examine some detailed representations
of algebras in \( V \circ S \).

MSC:
08B05  Equational logic, Mal’tsev conditions
08C15  Quasivarieties
08A05  Structure theory of algebraic structures

Keywords:
Mal’tsev product of varieties; semilattice sums; prolongation; Plonka sums; Lallement sums; regular and
irregular identities; regularization and pseudo-regularization of a variety

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References:
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