Năstăsescu, C.; Raianu, S.; Van Oystaeyen, F.
Modules graded by G-sets. (English) Zbl 0721.16025
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Let R be an associative ring with unity graded by a group G. By a G-set we mean a set A with a G-action on A. A left R-module N is said to be graded by A if $N = \oplus_{x\in A} N_x$ for some additive subgroups N_x such that $R_g N_x \subseteq N_{gx}$ for $g \in G$, $x \in A$. The aim of the paper is to study the category (G,A,R)-gr consisting of the left R-modules graded by a G-set A with degree preserving R-linear maps as morphisms. The inspiration comes from the paper of *E. Dade* [J. Reine Angew. Math. 369, 40-86 (1986; Zbl 0583.16001)], where some special cases of G-set gradations are applied to the Clifford theory of graded rings. In the first part of the paper the authors show that (G,A,R)-gr is a Grothendieck category. Then, the smash product construction R#G for a ring graded by a finite group G is extended to the case R#A of a finite G-set A. It is shown that (G,A,R)-gr is isomorphic to the category R#A-mod. Moreover, a matrix characterization of R#A is found. In the second part of the paper, certain functors introduced by Dade for G-sets of the form G/H (the left cosets of a subgroup H of G with G acting by left translations) are applied to general G-set gradations. These functors are particularly applied to the study of injective objects in (G,A,R)-gr.

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MSC:

16W50	Graded rings and modules (associative rings and algebras)
16D90	Module categories in associative algebras
16S40	Smash products of general Hopf actions
16D50	Injective modules, self-injective associative rings

Keywords:

injective module; G-action; category; left R-modules; G-set gradations; Clifford theory; graded rings; Grothendieck category; smash product; functors

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