Muhly, Paul S.; Solel, Baruch

Dilations for representations of triangular algebras. (English) Zbl 0721.46034

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Given a (not necessarily selfadjoint) subalgebra A of a unital C^*- algebra B and a contractive representation ρ of A on a Hilbert space $\mathcal{H}$, a dilation of ρ is a triple (π,V,$\mathcal{K}$), where π is a *-representation of B on a Hilbert space $\mathcal{K}$ and V is an isometric mapping $\mathcal{H}$ into $\mathcal{K}$ such that $\rho(a) = V^*\pi(a)V$ for all $a \in A$. A dilation is said to be minimal if the smallest subspace of $\mathcal{K}$ that reduces π and contains the range of V is $\mathcal{K}$ itself. This notion of dilation was introduced by *W. B. Arveson* [Acta Math. 123, 141-224 (1969; Zbl 0194.157)], where he showed that a dilation exists precisely when ρ is completely contractive.

In the present note, the authors consider the situation when B is a von Neumann algebra, A is σ-weakly closed, and ρ is σ- weakly continuous.

The main result of the paper can be stated as follows. Let M be a hyperfinite von Neumann algebra and let $\mathcal{T}$ be a σ-weakly closed subalgebra of M such that

(a) $\mathcal{T}$ is a σ-Dirichlet subalgebra of M (i.e. $A + A^*$ is σ-weakly dense in M);

(b) $\mathcal{T}$ contains a Cartan subalgebra of M (i.e. a subalgebra whose normalizer in M generates M and onto which there is a faithful normal expectation of M).

Then every σ-weakly continuous contractive representation of T has a unique (up to unitary equivalence) minimal dilation, which is a normal *-reprsentation of M.

The proof uses earlier work of *J. Feldman* and *C. Moore* [Trans. Am. Math. Soc. 234, 289-324 and 325-359 (1977; Zbl 0369.22009 and Zbl 0369.22010)], as well as work of the present authors and *K.-S. Saito* [Ann. of Math., II. Ser. 127, No.2, 245-278 (1988; Zbl 0649.47036)], giving a more concrete realization of M and $\mathcal{T}$. The paper ends with a result asserting that, if M and $\mathcal{T}$ are as above, ρ is a σ-weakly continuous representation of $\mathcal{T}$ on $\mathcal{H}$ with minimal dilation (π,V,$\mathcal{K}$), and S is an operator on $\mathcal{H}$ commuting with ρ ($\mathcal{T}$), then there is an operator $\tilde{S}$ on $\mathcal{K}$ commuting with π (M) such that $S = V^*\tilde{S}V$ and $\|\tilde{S}\| = \|S\|$.

Reviewer: T.A.Gillespie (Edinburgh)

MSC:

46L10	General theory of von Neumann algebras
47L30	Abstract operator algebras on Hilbert spaces
47A20	Dilations, extensions, compressions of linear operators
47A66	Quasitriangular and nonquasitriangular, quasidiagonal and nonquasidiagonal linear operators

Keywords:

contractive representation; dilation; *-representation; completely contractive; hyperfinite von Neumann algebra; σ -Dirichlet subalgebra; Cartan subalgebra; normalizer; faithful normal expectation; minimal dilation

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