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Homological stability for classical groups. (English) Zbl 07215117


Summary: “We prove a slope 1 stability range for the homology of the symplectic, orthogonal, and unitary groups with respect to the hyperbolic form, over any fields other than $\mathbb{F}_2$, improving the known range by a factor 2 in the case of finite fields. Our result more generally applies to the automorphism groups of vector spaces equipped with a possibly degenerate form (in the sense of Bak, Tits, and Wall). For finite fields of odd characteristic, and more generally fields in which $-1$ is a sum of two squares, we deduce a stability range for the orthogonal groups with respect to the Euclidean form, and a corresponding result for the unitary groups. In addition, we include an exposition of Quillen’s unpublished slope 1 stability argument for the general linear groups over fields other than $\mathbb{F}_2$, and use it to recover also the improved range of Galatius-Kupers-Randal-Williams in the case of finite fields, at the characteristic.”

From the introduction: “Our result in the case of symplectic, unitary, and orthogonal groups, follows from a more general stability result, stabilizing the automorphism group of any formed space with the hyperbolic form.”

For finite dimensional vector spaces $V_0 \subset V$ the authors define a relative building $\mathcal{T}(V, V_0)$ as the poset \{ $W < V$ | $W + V_0 = V$ \}. It is shown to be Cohen-Macaulay. They also prove vanishing of co-invariants of its top homology under the action of an appropriate group. Given $U < V_0$, the join construction provides a homotopy equivalence $\mathcal{T}(V, V_0) \to \mathcal{T}(V/U, V_0/U) \ast \mathcal{T}(V, U)$. These facts then need to be reproduced in the setting of formed spaces.

Next one follows Quillen and studies the spectral sequences arising from the actions of the groups on the buildings.

Reviewer: Wilberd van der Kallen (Utrecht)

MSC:

20J05 Homological methods in group theory
11E57 Classical groups

Keywords:

homology stability; symplectic group; orthogonal group; unitary group; formed space; hyperbolic form; relative building; Steinberg representation

Full Text: DOI

References:

[1] Ash, Avner; Putman, Andrew; Sam, Steven V., Homological vanishing for the Steinberg representation, Compos. Math., 154, 6, 1111-1130 (2018) · Zbl 06872524

Edited by FIZ Karlsruhe, the European Mathematical Society and the Heidelberg Academy of Sciences and Humanities
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