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A high order continuation method to locate exceptional points and to compute Puiseux series with applications to acoustic waveguides. (English) Zbl 1436.65040

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Summary: A numerical algorithm is proposed to explore in a systematic way the trajectories of the eigenvalues of non-Hermitian matrices in the parametric space and exploit this in order to find the locations of defective eigenvalues in the complex plane. These non-Hermitian degeneracies also called exceptional points (EP) have raised considerable attention in the scientific community as these can have a great impact in a variety of physical problems.

The method requires the computation of successive derivatives of two selected eigenvalues with respect to the parameter so that, after recombination, regular functions can be constructed. This algebraic manipulation permits the localization of exceptional points (EP), using standard root-finding algorithms and the computation of the associated Puiseux series up to an arbitrary order. This representation, which is associated with the topological structure of Riemann surfaces allows to efficiently approximate the selected pair in a certain neighborhood of the EP.

Practical applications dealing with guided acoustic waves propagating in straight ducts with absorbing walls and in periodic guiding structures are given to illustrate the versatility of the proposed method and its ability to handle large size matrices arising from finite element discretization techniques. The fact that EPs are associated with optimal dissipative treatments in the sense that they should provide best modal attenuation is also discussed.

MSC:

65F15 Numerical computation of eigenvalues and eigenvectors of matrices
76Q05 Hydro- and aero-acoustics

Keywords:

exceptional point; defective eigenvalue; Puiseux series; parametric eigenvalue problem; duct acoustics; bordered matrix

Software:

Eigtool; mpi4py; MUMPS; PETSc; petsc4py; SLEPc

Full Text: DOI

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