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Linear extension operators between spaces of Lipschitz maps and optimal transport. (English)

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The notion of K-gentle partition of unity is introduced in [*J. R. Lee* and *A. Naor*, Invent. Math. 160, 59–95 (2005; Zbl 1074.46004)] and the notion of K-Lipschitz retract is studied in [*S. I. Ohta*, Positivity 13, 407–425 (2009; Zbl 1198.54048)]. Several authors studied K-gentle partition of unity and K-Lipschitz retract [*A. Brudnyi* and *Y. Brudnyi*, Algebra Anal. 19, 106–118 (2007; Zbl 1213.54040); *P. Enflo*, Acta Math. 130, 309–317 (1973; Zbl 0267.46012); *G. Godefroy*, North-West. Eur. J. Math. 1, 1–6 (2015; Zbl 1386.46021); *G. Godefroy* and *N. Ozawa*, Proc. Amer. Math. Soc. 142, 1681–1687 (2014; Zbl 1291.46013); *J. Lindenstrauss* ,Michigan Math. J. 11, 263–287 (1964; Zbl 0195.42803); *A. Naor* and *Y. Rabani*, Israel J. Math. 219, 115–161 (2017; Zbl 1372.46020); *N. J. Kalton*, Collect. Math. 55, 171–217 (2004; Zbl 1069.46004)].

The principal objective in this paper is to study a weaker notion related to the Kantorovich-Rubinstein transport distance called K-random projection, and to show that K-random projections can still be used to provide linear extension operators for Lipschitz maps.

Reviewer: Lakehal Belarbi (Mostaganem)

MSC:

49Q22 Optimal transportation
47A20 Dilations, extensions, compressions of linear operators

Keywords:

infinite dimensional analysis; functions of bounded variation; open domains in Wiener spaces; geometric measure theory; Fomin differentiable measures

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