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On the existence of smooth orbital varieties in simple Lie algebras. (English) Zbl 07226671

Summary: Orbital varieties are the irreducible components of the intersection between a nilpotent orbit and a Borel subalgebra of the Lie algebra of a reductive group. There is a geometric correspondence between orbital varieties and irreducible components of Springer fibers. In type A, a construction due to Richardson implies that every nilpotent orbit contains at least one smooth orbital variety and every Springer fiber contains at least one smooth component. In this paper, we show that this property is also true for the other classical cases. Our proof uses the interpretation of Springer fibers as varieties of isotropic flags and van Leeuwen’s parametrization of their components in terms of domino tableaux. In the exceptional cases, smooth orbital varieties do not arise in every nilpotent orbit, as already noted by Spaltenstein. We however give a (nonexhaustive) list of nilpotent orbits which have this property. Our treatment of exceptional cases relies on an induction procedure for orbital varieties, similar to the induction procedure for nilpotent orbits.

MSC:

17B08 Coadjoint orbits; nilpotent varieties
05E16 Combinatorial aspects of groups and algebras
14L30 Group actions on varieties or schemes (quotients)
14M15 Grassmannians, Schubert varieties, flag manifolds
20G15 Linear algebraic groups over arbitrary fields

Full Text: DOI