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Boundedness of weighted iterated Hardy-type operators involving suprema from weighted Lebesgue spaces into weighted Cesàro function spaces. (English) [Zbl 07229051](#)

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Summary: The boundedness of the weighted iterated Hardy-type operators $T_{u,b}$ and $T_{u,b}^*$ involving suprema from weighted Lebesgue space $L_p(v)$ into weighted Cesàro function spaces $\text{Ces}_q(w, a)$ are characterized. These results allow us to obtain the characterization of the boundedness of the supremal operator R_u from $L^p(v)$ into $\text{Ces}_q(w, a)$ on the cone of monotone non-increasing functions. For the convenience of the reader, we formulate the statement on the boundedness of the weighted Hardy operator $P_{u,b}$ from $L^p(v)$ into $\text{Ces}_q(w, a)$ on the cone of monotone non-increasing functions. Under additional condition on u and b , we are able to characterize the boundedness of weighted iterated Hardy-type operator $T_{u,b}$ involving suprema from $L^p(v)$ into $\text{Ces}_q(w, a)$ on the cone of monotone non-increasing functions. At the end of the paper, as an application of obtained results, we calculate the norm of the fractional maximal function M_γ from $\Lambda^p(v)$ into $\Gamma^q(w)$.

MSC:

- 46E30 Spaces of measurable functions (L^p -spaces, Orlicz spaces, Köthe function spaces, Lorentz spaces, rearrangement invariant spaces, ideal spaces, etc.)
- 26D10 Inequalities involving derivatives and differential and integral operators
- 42B25 Maximal functions, Littlewood-Paley theory
- 42B35 Function spaces arising in harmonic analysis

Keywords:

weighted iterated Hardy operators involving suprema; Cesàro function spaces; fractional maximal functions; classical Lorentz spaces

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