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Proof of the Landau-Zener formula in an adiabatic limit with small eigenvalue gaps. (English)

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Summary: We consider a smooth operator-valued function H(t,δ) that has two isolated non-degenerate eigenvalues $E_{\mathcal{A}}(t,\delta)$ and $E_{\mathcal{B}}(t,\delta)$ for $\delta > 0$. We assume these eigenvalues are bounded away from the rest of the spectrum of H(t,δ), but have an avoided crossing with one another with a closest approach that is O(δ) as δ tends to zero. Under these circumstances, we study the small ϵ limit for the adiabatic Schrödinger equation $i\epsilon(\partial\psi/\partial t) = H(t,\epsilon^{1/2})\psi$.

We prove that the Landau-Zener formula correctly describes the coupling between the adiabatic states associated with the eigenvalues $E_{\mathcal{A}}(t,\delta)$ and $E_{\mathcal{B}}(t,\delta)$ as the system propagates through the avoided crossing.

MSC:

35Q40 PDEs in connection with quantum mechanics
81Q20 Semiclassical techniques, including WKB and Maslov methods applied to problems in quantum theory
35P99 Spectral theory and eigenvalue problems for partial differential equations

Keywords:
adiabatic Schrödinger equation; Landau-Zener formula

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