

Doyle, John R.; Silverman, Joseph H.

Moduli spaces for dynamical systems with portraits. (English) Zbl 1448.37132

Ill. J. Math. 64, No. 3, 375-465 (2020).

Summary: A *portrait* \mathcal{P} on \mathbb{P}^N is a pair of finite point sets $Y \subseteq X \subset \mathbb{P}^N$, a map $Y \rightarrow X$, and an assignment of weights to the points in Y . We construct a parameter space $\text{End}_d^N[\mathcal{P}]$ whose points correspond to degree d endomorphisms $f : \mathbb{P}^N \rightarrow \mathbb{P}^N$ such that $f : Y \rightarrow X$ is as specified by a portrait \mathcal{P} , and prove the existence of the GIT quotient moduli space $\mathcal{M}_d^N[\mathcal{P}] := \text{End}_d^N[\mathcal{P}] // \text{SL}_{N+1}$ under the SL_{N+1} -action $(f, Y, X)^\phi = (\phi^{-1} \circ f \circ \phi, \phi^{-1}(Y), \phi^{-1}(X))$ relative to an appropriately chosen line bundle. We also investigate the geometry of $\mathcal{M}_d^N[\mathcal{P}]$ and give two arithmetic applications.

MSC:

37P45 Families and moduli spaces in arithmetic and non-Archimedean dynamical systems

Cited in 1 Document

37P15 Dynamical systems over global ground fields

Keywords:

endomorphisms; quotient moduli space; weights

Software:

Magma; Multipliers-of-self-maps-on-P2

Full Text: [DOI](#) [Euclid](#)

References:

- [1] M. Arfeux, Compactification and trees of spheres covers, *Conform. Geom. Dyn.* 21 (2017), 225-246. · [Zbl 1370.37092](#)
- [2] B. Bielefeld, Y. Fisher, and J. Hubbard, The classification of critically preperiodic polynomials as dynamical systems, *J. Amer. Math. Soc.* 5 (1992), no. 2, 721-762. · [Zbl 0784.58031](#)
- [3] J. Blanc, J. K. Canci, and N. D. Elkies, Moduli spaces of quadratic rational maps with a marked periodic point of small order, *Int. Math. Res. Not. IMRN* 23 (2015), 12459-12489. · [Zbl 1349.37095](#)
- [4] W. Bosma, J. Cannon, and C. Playoust, The Magma algebra system, I: The user language, *J. Symbolic Comput.* 24 (1997), nos. 3-4, 235-265. · [Zbl 0898.68039](#)
- [5] T. Bousch, Sur quelques problèmes de dynamique holomorphe, Ph.D. dissertation, Université de Paris-Sud, Centre d'Orsay, 1992.
- [6] X. Buff, A. L. Epstein, and S. Koch, Irreducibility and postcritically finite unicritical polynomials, preprint, arXiv:1806.11221.
- [7] X. Buff and T. Lei, "The quadratic dynatomic curves are smooth and irreducible" in *Frontiers in Complex Dynamics*, Princeton Math. Ser. 51, Princeton Univ. Press, Princeton, 2014, 49-72. · [Zbl 1345.37050](#)
- [8] J. K. Canci and S. Vishkautsan, Quadratic maps with a periodic critical point of period 2, *Int. J. Number Theory* 13 (2017), no. 6, 1393-1417. · [Zbl 1392.37100](#)
- [9] K. Cordwell, S. Gilbertson, N. Nuechterlein, K. M. Pilgrim, and S. Pinella, On the classification of critically fixed rational maps, *Conform. Geom. Dyn.* 19 (2015), 51-94. · [Zbl 1384.37054](#)
- [10] L. DeMarco, The moduli space of quadratic rational maps, *J. Amer. Math. Soc.* 20 (2007), no. 2, 321-355. · [Zbl 1158.37020](#)
- [11] I. Dolgachev, *Lectures on Invariant Theory*, London Math. Soc. Lecture Note Ser. 296, Cambridge Univ. Press, Cambridge, 2003.
- [12] I. V. Dolgachev and Y. Hu, Variation of geometric invariant theory quotients, with an appendix by N. Ressayre, *Publ. Math. Inst. Hautes Études Sci.* 87 (1998), 5-56. · [Zbl 1001.14018](#)
- [13] A. Douady and J. H. Hubbard, Étude dynamique des polynômes complexes. Partie I, *Publ. Math. d'Orsay* 84, Univ. de Paris-Sud, Orsay, 1984. · [Zbl 0552.30018](#)
- [14] A. Douady and J. H. Hubbard, Étude dynamique des polynômes complexes. Partie II, *Publ. Math. d'Orsay* 85, Univ. de Paris-Sud, Orsay, 1985. · [Zbl 0552.30018](#)
- [15] A. Douady and J. H. Hubbard, A proof of Thurston's topological characterization of rational functions, *Acta Math.* 171 (1993), no. 2, 263-297. · [Zbl 0806.30027](#)

- [16] J. R. Doyle, Dynamical modular curves for quadratic polynomial maps, *Trans. Amer. Math. Soc.* 371 (2019), no. 8, 5655-5685. · [Zbl 1427.37071](#)
- [17] J. R. Doyle, H. Krieger, A. Obus, R. Pries, S. Rubinstein-Salzedo, and L. West, Reduction of dynatomic curves, *Ergodic Theory Dynam. Systems* 39 (2019), no. 10, 2717-2768. · [Zbl 07114045](#)
- [18] J. R. Doyle and B. Poonen, Gonality of dynatomic curves and strong uniform boundedness of preperiodic points, *Compos. Math.* 156 (2020), no. 4, 733-743. · [Zbl 1445.37080](#)
- [19] J. R. Doyle and J. H. Silverman, A uniform field-of-definition/field-of-moduli bound for dynamical systems on \mathbb{P}^N , *J. Number Theory* 195 (2019), 1-22. · [Zbl 1432.37124](#)
- [20] A. L. Epstein, Towers of finite type complex analytic maps, Ph.D. dissertation, City University of New York, 1993.
- [21] X. Faber and A. Granville, Prime factors of dynamical sequences, *J. Reine Angew. Math.* 661 (2011), 189-214. · [Zbl 1290.11019](#)
- [22] N. Fakhruddin, Questions on self maps of algebraic varieties, *J. Ramanujan Math. Soc.* 18 (2003), no. 2, 109-122. · [Zbl 1053.14025](#)
- [23] N. Fakhruddin, The algebraic dynamics of generic endomorphisms of \mathbb{P}^n , *Algebra Number Theory* 8 (2014), no. 3, 587-608. · [Zbl 1317.37116](#)
- [24] E. V. Flynn, B. Poonen, and E. F. Schaefer, Cycles of quadratic polynomials and rational points on a genus-2 curve, *Duke Math. J.* 90 (1997), no. 3, 435-463. · [Zbl 0958.11024](#)
- [25] Y. Gao, Preperiodic dynatomic curves for $(z \mapsto z^d + c)$, *Fund. Math.* 233 (2016), no. 1, 37-69. · [Zbl 1375.37133](#)
- [26] Y. Gao and Y. Ou, The dynatomic periodic curves for polynomial $(z \mapsto z^d + c)$ are smooth and irreducible, *Sci. China Math.* 57 (2014), no. 6, 1175-1192. · [Zbl 1336.37040](#)
- [27] D. Ghioca, K. Nguyen, and T. J. Tucker, Portraits of preperiodic points for rational maps, *Math. Proc. Cambridge Philos. Soc.* 159 (2015), no. 1, 165-186. · [Zbl 1371.11109](#)
- [28] P. Griffiths and J. Harris, *Principles of Algebraic Geometry*, reprint of the 1978 original, Wiley Classics Lib., Wiley, New York, 1994.
- [29] A. Guillot, Un théorème de point fixe pour les endomorphismes de l'espace projectif avec des applications aux feuilletages algébriques, *Bull. Braz. Math. Soc. (N.S.)* 35 (2004), no. 3, 345-362. · [Zbl 1085.58007](#)
- [30] A. Guillot, Semicompleteness of homogeneous quadratic vector fields, *Ann. Inst. Fourier (Grenoble)* 56 (2006), no. 5, 1583-1615. · [Zbl 1110.37040](#)
- [31] A. Guillot, Quadratic differential equations in three variables without multivalued solutions: Part I, *SIGMA Symmetry Integrability Geom. Methods Appl.* 14 (2018), paper no. 122.
- [32] A. Guillot and V. Ramírez, On the multipliers at fixed points of quadratic self-maps of the projective plane with an invariant line, *Comput. Methods Funct. Theory* 19 (2019), no. 4, 687-716. · [Zbl 1434.37027](#)
- [33] R. Hartshorne, *Algebraic Geometry*, Grad. Texts Math. 52, Springer, New York, 1977. · [Zbl 0367.14001](#)
- [34] B. Hutz, Dynatomic cycles for morphisms of projective varieties, *New York J. Math.* 16 (2010), 125-159. · [Zbl 1229.14022](#)
- [35] B. Hutz, Effectivity of dynatomic cycles for morphisms of projective varieties using deformation theory, *Proc. Amer. Math. Soc.* 140 (2012), no. 10, 3507-3514. · [Zbl 1302.37072](#)
- [36] B. Hutz and M. Tepper, Multiplier spectra and the moduli space of degree 3 morphisms on \mathbb{P}^1 , *J. Algebra Number Theory Appl.* 29 (2013), no. 2, 189-206. · [Zbl 1276.37051](#)
- [37] P. Ingram and J. H. Silverman, Primitive divisors in arithmetic dynamics, *Math. Proc. Cambridge Philos. Soc.* 146 (2009), no. 2, 289-302. · [Zbl 1242.11012](#)
- [38] G. Julia, Mémoire sur l'itération des fonctions rationnelles, *J. Math. Pures Appl.* 8 (1918), 47-246.
- [39] D. Krumm, Galois groups in a family of dynatomic polynomials, *J. Number Theory* 187 (2018), 469-511. · [Zbl 06866578](#)
- [40] D. Krumm, A finiteness theorem for specializations of dynatomic polynomials, *Algebra Number Theory* 13 (2019), no. 4, 963-993. · [Zbl 1442.37121](#)
- [41] E. Lau and D. Schleicher, Internal addresses in the Mandelbrot set and irreducibility of polynomials, Technical Report 1994/19, December 1994, <https://cds.cern.ch/record/448231>.
- [42] G. Levin, W. Shen, and S. van Strien, "Transversality for critical relations of families of rational maps: An elementary proof" in *New Trends in One-Dimensional Dynamics* 285, Springer Proc. Math. Stat., Springer, Cham, 2019, 201-220. · [Zbl 1446.37047](#)
- [43] A. Levy, The space of morphisms on projective space, *Acta Arith.* 146 (2011), no. 1, 13-31. · [Zbl 1285.37020](#)
- [44] A. Levy, M. Manes, and B. Thompson, Uniform bounds for preperiodic points in families of twists, *Proc. Amer. Math. Soc.* 142 (2014), no. 9, 3075-3088. · [Zbl 1401.37101](#)
- [45] A. Lins Neto, Fibers of the Baum-Bott map for foliations of degree two on \mathbb{P}^2 , *Bull. Braz. Math. Soc. (N.S.)* 43 (2012), no. 1, 129-169. · [Zbl 1347.37093](#)
- [46] M. Yu. Lyubich, Some typical properties of the dynamics of rational mappings, *Uspekhi Mat. Nauk* 38 (1983), no. 5(233), 197-198. · [Zbl 0546.58033](#)
- [47] R. Mañé, P. Sad, and D. Sullivan, On the dynamics of rational maps, *Ann. Sci. École Norm. Sup. (4)*, 16 (1983), no. 2, 193-217. · [Zbl 0524.58025](#)
- [48] M. Manes, Moduli spaces for families of rational maps on \mathbb{P}^1 , *J. Number Theory* 129 (2009), no. 7, 1623-1663. · [Zbl 1226.14017](#)

- [49] M. Manes and J. H. Silverman, A classification of degree 2 semi-stable rational maps $\mathbb{P}^2 \rightarrow \mathbb{P}^2$ with large finite dynamical automorphism group, *Ann. Fac. Sci. Toulouse Math.* (6) 28 (2019), no. 4, 733-811. · [Zbl 1432.37123](#)
- [50] C. T. McMullen, Families of rational maps and iterative root-finding algorithms, *Ann. Math.* (2), 125 (1987), no. 3, 467-493. · [Zbl 0634.30028](#)
- [51] J. S. Milne, *Étale Cohomology*, Princeton Math. Ser. 33, Princeton Univ. Press, Princeton, 1980.
- [52] J. Milnor, Geometry and dynamics of quadratic rational maps, with an appendix by the author and Lei Tan, *Exp. Math.* 2 (1993), no. 1, 37-83. · [Zbl 0922.58062](#)
- [53] J. Milnor, *Dynamics in One Complex Variable*, 3rd ed., Ann. of Math. Stud. 160, Princeton Univ. Press, Princeton, 2006. · [Zbl 1085.30002](#)
- [54] J. Milnor, “On Lattès maps” in *Dynamics on the Riemann Sphere*, Eur. Math. Soc., Zürich, 2006, 9-43. · [Zbl 1235.37015](#)
- [55] P. Morton, On certain algebraic curves related to polynomial maps, *Compos. Math.* 103 (1996), no. 3, 319-350. · [Zbl 0860.11065](#)
- [56] P. Morton and J. H. Silverman, Rational periodic points of rational functions, *Int. Math. Res. Not. IMRN* 2 (1994), 97-110. · [Zbl 0819.11045](#)
- [57] D. Mumford, J. Fogarty, and F. Kirwan, *Geometric Invariant Theory*, 3rd ed., *Ergeb. Math. Grenzgeb.* (2) 34, Springer, Berlin, 1994. · [Zbl 0797.14004](#)
- [58] C. Petsche, L. Szpiro, and M. Tepper, Isotriviality is equivalent to potential good reduction for endomorphisms of \mathbb{P}^N over function fields, *J. Algebra* 322 (2009), no. 9, 3345-3365. · [Zbl 1190.14013](#)
- [59] A. Poirier, Critical portraits for postcritically finite polynomials, *Fund. Math.* 203 (2009), no. 2, 107-163. · [Zbl 1179.37066](#)
- [60] B. Poonen, The classification of rational preperiodic points of quadratic polynomials over \mathbb{Q} : A refined conjecture, *Math. Z.* 228 (1998), no. 1, 11-29. · [Zbl 0902.11025](#)
- [61] M. Rees, A partial description of parameter space of rational maps of degree two, I, *Acta Math.* 168 (1992), nos. 1-2, 11-87. · [Zbl 0774.58035](#)
- [62] M. Rees, A partial description of the parameter space of rational maps of degree two, II, *Proc. London Math. Soc.* (3) 70 (1995), no. 3, 644-690. · [Zbl 0827.58048](#)
- [63] J. Silverman, Good reduction and Shafarevich-type theorems for dynamical systems with portrait level structures, *Pacific J. Math.* 295 (2018), no. 1, 145-190. · [Zbl 1396.37097](#)
- [64] J. H. Silverman, The space of rational maps on \mathbb{P}^1 , *Duke Math. J.* 94 (1998), no. 1, 41-77. · [Zbl 0966.14031](#)
- [65] J. H. Silverman, *The Arithmetic of Dynamical Systems*, *Grad. Texts Math.* 241, Springer, New York, 2007. · [Zbl 1130.37001](#)
- [66] J. H. Silverman, *Moduli Spaces and Arithmetic Dynamics*, *CRM Monogr. Ser.* 30, Amer. Math. Soc. Providence, RI, 2012. · [Zbl 1247.37004](#)
- [67] T. Ueda, “Complex dynamics on projective spaces—Index formula for fixed points” in *Dynamical Systems and Chaos*, Vol. 1 (Hachioji, 1994), World Sci., River Edge, NJ, 1995, 252-259. · [Zbl 0991.32504](#)
- [68] L. W. West, The moduli space of rational maps, Ph.D. dissertation, City University of New York, 2015.
- [69] L. W. West, The moduli space of cubic rational maps, preprint, arXiv:1408.3247.

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.