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The extensions of convergence rates of Kaczmarz-type methods. (English) Zbl 1452.65062
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Summary: Kaczmarz-type methods, such as the randomized Kaczmarz method, the block Kaczmarz method and the Cimmino method, can be derived from the Kaczmarz method. In this paper, we introduce a new error term $\|x_k - P_{N(A)}x_0 - x^\dagger\|_2$ for Kaczmarz-type methods, where x^\dagger is the generalized solution of $Ax = b$ and $P_{N(A)}x_0$ is the orthogonal projection of a given initial value x_0 onto the null space $N(A)$. It includes the well-known error term $\|x_k - x^*\|_2$ as a special case when $x_0 = 0$ and $x^\dagger = x^*$, where x^* is a true solution of $Ax = b$. We investigate the behavior of the new error term and establish the corresponding convergence rates for Kaczmarz-type methods. Especially, from the estimate of new error term for the Kaczmarz method, we can get a more simple proof for the convergence of the Kaczmarz method.

MSC:

- 65F10 Iterative numerical methods for linear systems
- 65F08 Preconditioners for iterative methods
- 65N22 Numerical solution of discretized equations for boundary value problems involving PDEs
- 65J20 Numerical solutions of ill-posed problems in abstract spaces; regularization

Keywords:

Kaczmarz-type methods; convergence rate; error estimates

Software:

Regularization tools

Full Text: [DOI](#)

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