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Local Gorenstein duality for cochains on spaces. (English) [Zbl 1451.55007](#)
J. Pure Appl. Algebra 225, No. 2, Article ID 106495, 23 p. (2021).

The paper under review develops a homotopical notion of local Gorenstein duality for commutative ring spectra. The main result provides conditions under which a ring spectrum $R \rightarrow k$ has local Gorenstein duality. The method of proof uses an ascent theorem for the local case based on the general Gorenstein ascent of *W. G. Dwyer* et al. [*Adv. Math.* 200, No. 2, 357–402 (2006; [Zbl 1155.55302](#))]. A number of examples of the form $R = C^*(X; k)$ are given, for various spaces X of interest.

Reviewer: [Niles Johnson \(Newark\)](#)

MSC:

- [55U30](#) Duality in applied homological algebra and category theory (aspects of algebraic topology)
- [55R35](#) Classifying spaces of groups and H -spaces in algebraic topology
- [13H10](#) Special types (Cohen-Macaulay, Gorenstein, Buchsbaum, etc.)
- [13D45](#) Local cohomology and commutative rings

Keywords:

[Gorenstein duality](#); [local cohomology](#); [structured ring spectra](#); [\$p\$ -compact groups](#); [\$p\$ -local finite groups](#)

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