
Killing forms in this article are studied on simply connected 2-step nilpotent Lie groups endowed with left-invariant Riemannian metrics, and they are classified when the center of the Lie group is at most two-dimensional. Initially, here some general results are proved. It is shown that the de Rham decomposition of nilpotent Lie groups is closely related to the decomposition of the underlying metric Lie algebra into an orthogonal direct sum of ideals. Also, it is shown that every left-invariant Killing form on a product of Riemannian Lie groups is a sum of Killing forms on the factors and a parallel form. Further it is proved that, if \( n \) is a non-abelian nilpotent Lie algebra and its center is one-dimensional, then \( n \) is isomorphic to the Heisenberg Lie algebra \( h_{2n+1} \) and its metric structure can be described by an invertible matrix in \( \text{so}(2n) \). Then it is proved that, if \( (n, g) \) is a 2-step nilpotent metric Lie algebra with two-dimensional center, then, if \( (n, g) \) is irreducible, the space \( \mathcal{K}^p(n, g) \) of Killing \( p \)-forms on \( (n, g) \) has the following properties: \( \mathcal{K}^1(n, g) \) is two-dimensional; \( \mathcal{K}^2(n, g) \) is one-dimensional if \( n \) has a bi-invariant \( g \)-orthogonal complex structure, and zero otherwise; \( \mathcal{K}^p(n, g) = 0 \) for \( 4 \leq p \leq \dim(n) - 1 \) and \( \mathcal{K}^p(n, g) \) one-dimensional if \( p = \dim(n) \). A similar description is given if \( (n, g) \) is reducible.

Reviewer: V. V. Gorbatevich (Moskva)

MSC:
- 53C30 Differential geometry of homogeneous manifolds
- 53C20 Global Riemannian geometry, including pinching
- 53D25 Geodesic flows in symplectic geometry and contact geometry
- 22E25 Nilpotent and solvable Lie groups
- 53C15 General geometric structures on manifolds (almost complex, almost product structures, etc.)
- 53C21 Methods of global Riemannian geometry, including PDE methods; curvature restrictions

Keywords:
Killing forms; 2-step nilpotent Lie groups; irreducible Lie algebras

Full Text: DOI

References:


This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.