Summary: We study existence of invariant Einstein metrics on complex Stiefel manifolds $G/K = \text{SU}(\ell + m + n)/\text{SU}(n)$ and the special unitary groups $G = \text{SU}(\ell + m + n)$. We decompose the Lie algebra $\mathfrak{g}$ of $G$ and the tangent space $\mathfrak{p}$ of $G/K$, by using the generalized flag manifolds $G/H = \text{SU}(\ell + m + n)/\text{SU}(\ell) \times \text{SU}(m) \times \text{SU}(n)$ We parametrize scalar products on the 2-dimensional center of the Lie algebra of $H$, and we consider $G$-invariant and left invariant metrics determined by $\text{Ad}(\text{SU}(\ell) \times \text{SU}(m) \times \text{SU}(n))$-invariant scalar products on $\mathfrak{g}$ and $\mathfrak{p}$ respectively. Then we compute their Ricci tensor for such metrics. We prove existence of $\text{Ad}(\text{SU}(1) \times \text{SU}(2) \times \text{SU}(2))$-invariant Einstein metrics on $V_2\mathbb{C}^5 = \text{SU}(5)/\text{SU}(2)$, $\text{Ad}(\text{SU}(2) \times \text{SU}(2) \times \text{SU}(2))$-invariant Einstein metrics on $V_2\mathbb{C}^6 = \text{SU}(6)/\text{SU}(2)$, and $\text{Ad}(\text{SU}(n) \times \text{SU}(n))$-invariant Einstein metrics on $V_{2m}\mathbb{C}^{2m+n} = \text{SU}(2m+n)/\text{SU}(n)$. We also prove existence of $\text{Ad}(\text{SU}(1) \times \text{SU}(2) \times \text{SU}(2))$-invariant Einstein metrics on the compact Lie group $\text{SU}(5)$, which are not naturally reductive. The Lie group $\text{SU}(5)$ is the special unitary group of smallest rank known for the moment, admitting non naturally reductive Einstein metrics. Finally, we show that the compact Lie group $\text{SU}(4+n)$ admits two non naturally reductive $\text{Ad}(\text{SU}(2) \times \text{SU}(2) \times \text{SU}(n))$-invariant Einstein metrics for $2 \leq n \leq 25$, and four non naturally reductive Einstein metrics for $n \geq 26$. This extends previous results of K. Mori about non naturally reductive Einstein metrics on $\text{SU}(4+n)(n \geq 2)$.

MSC:

| 53C25 | Special Riemannian manifolds (Einstein, Sasakian, etc.) |
| 53C30 | Differential geometry of homogeneous manifolds |
| 13P10 | Gröbner bases; other bases for ideals and modules (e.g., Janet and border bases) |
| 65H10 | Numerical computation of solutions to systems of equations |
| 68W30 | Symbolic computation and algebraic computation |

Keywords:

homogeneous space; Einstein metric; Stiefel manifold; special unitary group; invariant metric; isotropy representation; Gröbner basis

Full Text: DOI Euclid

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