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Can mathematics be proved consistent? Gödel's shorthand notes & lectures on incompleteness. (English) Zbl 1466.03001

Sources and Studies in the History of Mathematics and Physical Sciences. Cham: Springer (ISBN 978-3-030-50875-3/hbk; 978-3-030-50878-4/pbk; 978-3-030-50876-0/ebook). ix, 263 p. (2020).

The book under review, published in the Springer series of *Sources and Studies in the History of Mathematics and Physical Sciences* (since 1975), "contains all that is found in Gödel's preserved shorthand notebooks on his research that led to the famous incompleteness theorems of formal systems. The notes are followed by the original version of his article, before a dramatic change just a few days after it was handed in for publication, and six lectures and seminars in consequence of his celebrated result published in 1931" (p. v). "The Kurt Gödel papers on incompleteness that this book explores are kept at the Firestone Library of Princeton University" (p. vii). The title of the book (*Can mathematics be proved consistent*?) is actually the title of the lecture notes that "were founded in June 2019 by Maria Hämeen-Anttila, during a search of the manuscript for Gödel's 30 December 1933 lecture *The present situation in the foundations of mathematics*, the final version of which is published in the third volume of *K. Gödel*'s [Collected works. Volume III: Unpublished essays and lectures. Edited by Solomon Feferman and others. Oxford: Oxford University Press (2001; Zbl 1074.01015)]" (p. 56); the lecture notes appear in the very last section of the book.

Among the very many interesting historical facts about and around the discovery of the incompleteness theorems, discussed in this fascinating book, are the following couple of instances:

– *Gödel first considered sound theories (and the notion of ω-consistency was introduced later).*

Actually, no "trace of Gödel's original proof of the incompleteness theorems that uses a truth definition is left in his published article" (p. 12). "After the Königsberg meeting, the concept of truth and even the intuitive notion of 'correctness' doesn't even find these words anymore" (p. 13). One can see in the book that Gödel's assumptions on the theories subject to the incompleteness theorems included:

"2. All provable propositions are true" (p. 61),

"1·) Each proposition provable in it is true" (p. 65), or

"II Each system that arises through the adjunction of a definable class of axioms to the system S and that is $\aleph_0$-consistent" (p. 76).

"At the end of the introduction to his article, Gödel writes about the two conditions his result needs: first, that the concept of 'provable formula' can be defined within a system, and, secondly, that 'each provable formula be contentfully correct.' The comment is that 'the exact carrying through of the above proof has as its main task to avoid completely the contentful interpretation of the formulas of the system considered.' This has been changed into: 'has as its tasks among others the substitution of the second of the conditions presented by a purely formal and much weaker one"' (p. 49). The much weaker condition was later called "ω-consistency" (evolved from the more set-theoretic looking "$\aleph_0$-consistency") by Gödel.

– *Gödel's correspondence with von Neumann and the* "tricky change of his manuscript after it had been submitted for publication" (p. v).

Indeed, among "Gödel's audience in Königsberg sat Johann von Neumann who reacted at once and wanted more explanations. The two had discussions at the conference and in Berlin, where Gödel stayed for a few days immediately after the conference" (p. 13). "In Gödel's first letter draft, he wants to assure von Neumann that he had both results, even mentioning Carnap as witness and quoting 17 September as the date he gave his October 1930 note to Hans Hahn who would communicate it to the Academy, and a promise to send a copy of the part of the manuscript that deals with the second theorem. Then come eight pages of attempts at a satisfactory formulation, and the second letter draft in which just the proofs of the incompleteness article are promised once they arrive" (p. 25). Gödel wanted to write those to von Neumann because he "panicked at the prospect of von Neumann publishing his second theorem" (p. 26). Thus, a "shadow is cast on Gödel's great achievement; [there] is no way of undoing the fact that Gödel together with Hahn played a well-planned trick to persuade von Neumann not to publish [the second

incompleteness theorem]" (p. 25).

The book is very well-written, historically, mathematically, and philosophically. I could spot only two typos:

(i) On page 174 line 14 there is an indication of a footnote numbered "6)" but there is no footnote 6) on that page (or anywhere else); the footnotes of page 174 are 5), 7), 8), and 9).

(ii) On page 251 line 4 the opened parenthesis before the word "impossibility" is never closed.

I strongly recommend reading this book for anyone interested in the incompleteness phenomenon, which is one of the greatest achievements of science in the 20th century.

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MSC:

03-03	History of mathematical logic and foundations
03F40	Gödel numberings and issues of incompleteness
01A75	Collected or selected works; reprintings or translations of classics
01A70	Biographies, obituaries, personalia, bibliographies
01A60	History of mathematics in the 20th century

Keywords:

incompleteness; consistency; Gödel

Biographic references:

Gödel, Kurt

Full Text: DOI