Koenigsmann, Jochen

Decidability in local and global fields. (English) Zbl 1458.11171

Sirakov, Boyan (ed.) et al., Proceedings of the international congress of mathematicians, ICM 2018, Rio de Janeiro, Brazil, August 1–9, 2018. Volume II. Invited lectures. Hackensack, NJ: World Scientific; Rio de Janeiro: Sociedade Brasileira de Matemática (SBM). 45-59 (2018).

This is a survey of the research on the decidability of the first-order theories of local and global fields. One well-known question here concerns Hilbert's 10th problem, which asks for an algorithm that for a given $f \in \mathbb{Z}[x_1, \ldots, x_n]$ as input decides whether it has a root in $\mathbb{Z}^n$ or not. The author notes that this can be generalized to any ring R (which are taken to be integral domain here) in two ways: (1) when the second occurrence of $\mathbb{Z}$ above is replaced by R (denoted by $\mathbf{H10}/R$), and (2) when both occurrences of $\mathbb{Z}$ above are replaced by R (denoted by $\mathbf{H10}^+/R$). One celebrated open problem is the algorithmic solvability of $\mathbf{H10}/\mathbb{Q}$. Local fields of positive characteristic are studied in Section 2, and global fields in Section 3; the last Section 4 considers two infinite extensions of $\mathbb{Q}_p$.

The paper collects an up-to-date state of the art of the results and open problems in the field. It is a good read despite of some minor misprints (such as the references [P09a], [Poo02], and [MR10] on page 50 and in Theorems 3.7 and 3.8, respectively) whose intended meanings can be easily guessed.

For the entire collection see [Zbl 1437.00044].

Reviewer: Saeed Salehi (Tabriz)

MSC:

11U05	Decidability (number-theoretic aspects)
03B25	Decidability of theories and sets of sentences
11F85	p-adic theory, local fields
12J10	Valued fields
12L05	Decidability and field theory

Keywords:

decidability; local fields; global fields; Hilbert's 10th problem

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