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Directly decomposable ideals and congruence kernels of commutative semirings. (English)
Zbl 07254887

Summary: As pointed out in the monographs *J. S. Golan* [Semirings and their applications. Dordrecht: Kluwer Academic Publishers (1999; Zbl 0947.16034)], *W. Kuich* and *A. Salomaa* [Semirings, automata, languages, EATCS Monographs on Theoretical Computer Science, 5. Berlin, Springer-Verlag (1986; Zbl 0582.68002)] on semirings, ideals play an important role despite the fact that they need not be congruence kernels as in the case of rings. Hence, having two commutative semirings S_1 and S_2, one can ask whether an ideal I of their direct product $S = S_1 \times S_2$ can be expressed in the form $I_1 \times I_2$, where I_j is an ideal of S_j for $j = 1, 2$. Of course, the converse is elementary, namely if I_j is an ideal of S_j for $j = 1, 2$ then $I_1 \times I_2$ is an ideal of $S_1 \times S_2$. Having a congruence Θ on a commutative semiring S, its 0-class is an ideal of S, but not every ideal is of this form. Hence, the lattice IdS of all ideals of S and the lattice $\mathrm{Ker}S$ of all congruence kernels (i.e. 0-classes of congruences) of S need not be equal. Furthermore, we show that the mapping $\Theta \to [0]\Theta$ need not be a homomorphism from $\mathrm{Con}S$ onto $\mathrm{Ker}S$. Moreover, the question arises when a congruence kernel of the direct product $S_1 \times S_2$ of two commutative semirings can be expressed as a direct product of the corresponding kernels on the factors. In the paper we present necessary and sufficient conditions for such direct decompositions both for ideals and for congruence kernels of commutative semirings. We also provide sufficient conditions for varieties of commutative semirings to have directly decomposable kernels.

MSC:

16Y60	Semirings
08A05	Structure theory of algebraic structures
08B10	Congruence modularity, congruence distributivity
08A30	Subalgebras, congruence relations

Keywords:

semiring; congruence; ideal; skew ideal; congruence kernel; direct decomposability

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