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Dispersive rarefaction wave with a large initial gradient. (English) Zbl 1448.35442
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Summary: Consider the Cauchy problem for the Korteweg-de Vries equation with a small parameter at the highest derivative and a large gradient of the initial function. Numerical and analytical methods show that the obtained using renormalization formal asymptotics, corresponding to rarefaction waves, is an asymptotic solution of the KdV equation. The graphs of the asymptotic solutions are represented, including the case of non-monotonic initial data.

MSC:

- 35Q53 KdV equations (Korteweg-de Vries equations)
- 35B40 Asymptotic behavior of solutions to PDEs
- 35C20 Asymptotic expansions of solutions to PDEs
- 35B25 Singular perturbations in context of PDEs
- 65D32 Numerical quadrature and cubature formulas

Keywords:

Korteweg-de Vries; Cauchy problem; asymptotic behavior; rarefaction wave

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