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Computational finite element model for surface wrinkling of shells on soft substrates. (English) Zbl 07264491

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Summary: We provide a robust finite element formulation for quantitative prediction of surface wrinkling of pressurized elastic shells on soft substrates. Our theory is build on three basic assumptions which involve thin shell kinematics, the approximation of the substrate response by a Winkler foundation and a model order reduction of the displacement field. Our element keeps all the nonlinear terms of the reduced model. The proposed formulation does not require any perturbations, either in the initial geometry or in the load, to incite the transition from fundamental to secondary equilibrium path for the considered set of shells, due to inherent asymmetric imperfections in the mesh. Numerical simulations using the derived element and an advanced path-following method on full spheres, hemispheres and spheroids show a very good quantitative agreement with theoretical predictions and experiments on the characteristic wavelength of the pattern as well as the qualitative depiction of the pattern evolution.

MSC:

68W Algorithms in computer science
74 Mechanics of deformable solids
74S Numerical and other methods in solid mechanics

Keywords:

surface wrinkling; film-substrate composite; model order reduction; thin shells

Software:

AceFEM

Full Text: DOI

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