

Cioffi, Francesca

The range of all regularities for polynomial ideals with a given Hilbert function. (English)

[Zbl 07268500](#)

J. Algebra 566, 435-442 (2021).

Let A be a polynomial ring over a field K and let I denote any homogeneous ideal of A . This paper presents a proof of the following statement suggested by Le Tuan Hoa in a private communication,

Theorem 1: Given an Hilbert function u , the set $\{reg(I) : A/I \text{ has Hilbert function } u\}$ is an interval of integers.

Since for every homogenous ideal I , there exists a strongly stable ideal J such that $reg(I) = reg(J)$ and $reg(J)$ is the highest degree, denoted by $\nu(J)$, of a minimal generator of J , the theorem 1 is equivalent to the following one,

Theorem 2: Given an Hilbert function u , the set $\{\nu(J) : J \text{ is strongly stable and } A/J \text{ has Hilbert function } u\}$ is an interval of integers.

The proof is based on constructive arguments presented in the paper [*F. Cioffi et al., Exp. Math.* 24, No. 4, 424–437 (2015; [Zbl 1333.13028](#))], mainly on a construction called *expanded lifting*, that allows, given a saturated homogeneous ideal I with Hilbert function $\preceq u$, to create a homogenous ideal J with Hilbert function u .

Reviewer: [Gema Maria Diaz Toca \(Murcia\)](#)

MSC:

- [13P10](#) Gröbner bases; other bases for ideals and modules (e.g., Janet and border bases)
- [14Q99](#) Computational aspects in algebraic geometry
- [68W30](#) Symbolic computation and algebraic computation
- [11Y55](#) Calculation of integer sequences
- [13C99](#) Theory of modules and ideals in commutative rings

Keywords:

[Castelnuovo-Mumford regularity](#); [Hilbert function](#); [minimal function](#); [strongly stable ideal](#)

Software:

[CoCoA](#)

Full Text: [DOI](#)

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