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Constructing cocyclic Hadamard matrices of order $4p$. (English) [Zbl 1451.05032]

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Summary: Cocyclic Hadamard matrices (CHMs) were introduced by W. de Launey and K. J. Horadam [Des. Codes Cryptography 3, No. 1, 75–87 (1993; Zbl 0838.05019)] as a class of Hadamard matrices (HMs) with interesting algebraic properties. P. Ó Catháin and M. Röder [Des. Codes Cryptography 58, No. 1, 73–88 (2011; Zbl 1246.05033)] described a classification algorithm for CHMs of order $4n$ based on relative difference sets in groups of order $8n$; this led to the classification of all CHMs of order at most 36. On the basis of work of W. de Launey and D. Flannery [Algebraic design theory. Providence, RI: American Mathematical Society (AMS) (2011; Zbl 1235.05001)], we describe a classification algorithm for CHMs of order $4p$ with $p$ a prime; we prove refined structure results and provide a classification for $p \leq 13$. Our analysis shows that every CHM of order $4p$ with $p \equiv 1 \pmod{4}$ is equivalent to a HM with one of five distinct block structures, including Williamson-type and (transposed) Ito matrices. If $p \equiv 3 \pmod{4}$, then every CHM of order $4p$ is equivalent to a Williamson-type or (transposed) Ito matrix.

MSC:

05B20 Combinatorial aspects of matrices (incidence, Hadamard, etc.)
05B10 Combinatorial aspects of difference sets (number-theoretic, group-theoretic, etc.)

Keywords:
cocyclic development; Hadamard matrix; Ito type; Williamson type

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