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Free poles of quadratic differentials and non-overlapping domains. (Russian) Zbl 0727.30019

Questions of analysis and approximation, Collect. Sci. Works, Kiev, 23-26 (1989).

[For the entire collection see Zbl 0685.00003.]

Let $\mathfrak{M}_n$ be the class of n-tuples $F_n = (f_1, ..., f_n)$ of functions which are meromorphic and one-to-one in the unit disc D fulfilling the following properties:

(i) $E_i \cap E_j = \emptyset$ for $i \neq j$, $i, j = 1, 2, ..., n$, where $E_k = f_k(D)$.

(ii) $f_k(0) = a_k$, $k = 1, 2, ..., n$, where $a_k \in E$ are given and fulfill the condition $(1/\mathrm{n})\sum_{k=1}^n |a_k| = 1$.

The following extremal problem in the class $\mathfrak{M}_n$ is studied: find $I_n = \max_{F_n \in \mathfrak{M}_n} \prod_{k=1}^n |f'_k(0)|$. Theorem 1. For every extremal n-tuple $\psi_n = (\psi_1, \psi_2, ..., \psi_n) \in \mathfrak{M}_n$ the set $\mathbb{C} \cup \{\infty\} \setminus (\cup_{k=1}^n E_k)$, where $E_k = \psi_k(D)$, is composed from a finite number of critical points and trajectories of the quadratic differential

$$Q(w)dw^2 = -\sum_{k=1}^n \left(\frac{1}{(a_k - w)^2} + \frac{|a_k|}{w(a_k - w)}dw^2 \text{ and } (\cup_{k=1}^n E_k) \cup \{\infty\} = \bar{\mathbb{C}}.\right.$$

Theorem 2. For $n = 3$, $I_3 = 32/3\sqrt{3}$. The extremal 3-tuple ψ_3 is described explicitely.

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