Bakhtin, A. K.

Some extremal problems for multiply connected domains. (Russian) Zbl 0727.30020
Questions of analysis and approximation, Collect. Sci. Works, Kiev, 19-22 (1989).

[For the entire collection see Zbl 0685.00003.]

Let $E_n \subset \bar{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$ by an n-times connected region containing $z = \infty$ and bounded by n circles. Let denote Σ_n the class of functions meromorphic in E_n, which are one-to-one in E_n and of the form $f(z) = \epsilon z + \alpha_0 + \alpha_1/z$, $|\epsilon| = d$, in the neighborhood of $z = \infty$. Let $\bar{\mathbb{C}} \setminus f(E_n) = \cup_{i=1}^{n} K_i$, where K_i are mutually disjoint continua. Let denote

$$d_n(f) = \max_{c_i \in K_i} [\prod_{1 \le i < j \le n} |c_i - c_j|^{2/(n(n-1))}].$$

The following extremal problem is studied: to find $\max_{f \in \Sigma_n} d_n(f)$.

Theorem. The compact solving the extremal problem is composed from n mutually disjoint continua K_i, $i = 1, 2, ..., n$, each of them being the closure of a finite number of arcs of trajectories of the quadratic differential

$$Q(w)dw^2 = -\sum \frac{dw^2}{(c_i^0 - w)(c_j^0 - w)}.$$

The case $n = 3$ is discussed in detail.

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MSC:

30C70 Extremal problems for conformal and quasiconformal mappings, variational methods

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