Fonda, A.; **Gossez, J.-P.**

Semicoercive variational problems at resonance: An abstract approach. (English)

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Differ. Integral Equ. 3, No. 4, 695-708 (1990).

The authors consider the Dirichlet problem

$$(*) \quad -\Delta u - \lambda_1 u + g(x,u) = 0 \text{ in } \Omega, \quad u = 0 \text{ on } \partial\Omega,$$

where $\Omega \subset \mathbb{R}^N$ (N$\geq$ 1) is bounded and λ_1 is the first eigenvalue of (-Δ) on $H_0^1(\Omega)$. The Carathéodory function g: $\Omega \times \mathbb{R} \to \mathbb{R}$ satisfies

$$|g(x,u)| \leq a|u|^{q-1} + b(x),$$

where $q < \infty$ if $N = 2$, $q < 2N/(N-2)$ if $N \geq 3$, and $b(x) \in L^{q'}(\Omega)$, $1/q + 1/q' = 1$. In case $N = 1$ it is assumed that for any $r > 0$, $\sup_{|u| \leq r} |g(x,u)| \in L^1(\Omega)$.

The associated functional to problem (*)

$$F(u) = 1/2 \int_\Omega [|\nabla u|^2 - \lambda_1 |u|^2] dx + \int_\Omega \int_{s=0}^u g(x,s) ds dx =: A(u) + B(u)$$

is a weakly lower semicontinuous C^1 functional on H_0^1, whose critical points are the weak solutions of (*). If F(u) is coercive, i.e. F(u)$\to \infty$ as $\|u\| \to \infty$ in H_0^1, then F has a minimum and consequently (*) has a weak solution. The authors mainly study the coercivity of functionals $F = A + B$ where A is semicoercive with respect to a subspace and B is coercive on a complementary subspace. Applications are given to the existence of solutions of problem (*).

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MSC:

35J65 Nonlinear boundary value problems for linear elliptic equations

49J20 Existence theories for optimal control problems involving partial differential equations

35J15 Second-order elliptic equations

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semilinear Dirichlet problem; coercivity; existence