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Unusual extension-torsion-inflation couplings in pressurized thin circular tubes with helical anisotropy. (English) Zbl 07273334

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Summary: We present a thin tube formulation for coupled extension-torsion-inflation deformation in helically reinforced pressurized circular tubes. Both compressible and incompressible tubes are considered. On applying the thin tube limit, the nonlinear ordinary differential equation to obtain the in-plane radial displacement is converted into a set of two simple algebraic equations for the compressible case and one equation for the incompressible case. This allows us to obtain analytical expressions, in terms of the tube's intrinsic twist, material constants, and the applied pressure, which can predict whether such tubes would overwind/unwind on being infinitesimally stretched or exhibit positive/negative Poisson's effect. We further show numerically that such tubes can be tuned to generate initial overwinding followed by rapid unwinding as observed during finite stretching of a torsionally relaxed DNA. Finally, we demonstrate that such tubes can also exhibit usual deflation initially followed by unusual inflation as the tube is finitely stretched.

MSC:

74 Mechanics of deformable solids

Keywords:

effective Poisson's ratio; extension-torsion coupling; helical anisotropy; negative Poisson's effect; thin tube

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