Summary: The most common problems in nature are about non-conservative non-linearity. Non-conservative non-linear problems can be studied with variational problems of non-standard Lagrangians. Birkhoffian mechanics, as an extension of Hamiltonian mechanics naturally, is a sign that analytical mechanics has entered a new stage of development. Therefore, the study of dynamics based on non-standard Birkhoffians provides a new idea for solving non-conservative nonlinear dynamics problems. In this paper the dynamics models based on non-standard Birkhoffians, including exponential Birkhoffian, power law Birkhoffian, and logarithm Birkhoffian, are proposed, which are called non-standard Birkhoffian systems. Firstly, the Pfaff-Birkhoff principles with non-standard Birkhoffians are established, the differential equations of motion of non-standard Birkhoffian dynamics are also derived. Secondly, in accordance with the invariance of Pfaff action under the infinitesimal transformations, giving the definitions and criteria of Noether symmetric and quasi-symmetric transformations of non-standard Birkhoffian dynamics. And next, Noether’s theorems for non-standard Birkhoffian dynamics are proved, and the connections between Noether symmetries and conserved quantities of non-standard Birkhoffian dynamics are established; Finally, three examples are given to illustrate the applications of the results.

MSC:
70H30 Other variational principles in mechanics
70G75 Variational methods for problems in mechanics
70H33 Symmetries and conservation laws, reverse symmetries, invariant manifolds and their bifurcations, reduction for problems in Hamiltonian and Lagrangian mechanics
37J51 Action-minimizing orbits and measures for finite-dimensional Hamiltonian and Lagrangian systems; variational principles; degree-theoretic methods

Keywords: non-standard Birkhoffian dynamics; Pfaff-Birkhoff principles; Noether’s theorem; conserved quantity

Full Text: DOI

References:
[12] Musielak ZE, Davachi N. Rosario-franco m. Lagrangian formalism and its auxiliary conditions: special function equations and...


[22] Lutzky, M., Dynamical symmetries and conserved quantities, J Phys A: Math Gen, 12, 7, 973-981 (1979) · Zbl 0413.70013


[33] Birkhoff, G. D., Dynamical systems (1927), Providence RI: AMS College Publ · Zbl 53.0733.03

[34] Santilli, R. M., Foundations of theoretical mechanics (1983), Springer New York · Zbl 1013.34004


