Burkett, Shawn T.; Lewis, Mark L.
A Frobenius group analog for Camina triples. (English) [Zbl 07285334]

If $G$ is a (finite) Frobenius group, then there are $N \trianglelefteq G$ and $H \leq G$ such that $G = NH$, $([N], [H]) = 1$ and $C_G(x) \leq N$ for all $x \in N \setminus \{1\}$. In particular, every element of $G \setminus N$ acts fixed-point-freely on $N$. One key property of Frobenius groups is that the conjugacy class of any element $g \in G \setminus N$ is a union of cosets of $N$. A pair $(G, N)$ is called a Camina pair if $N$ is a proper, nontrivial normal subgroup of $G$ where every element $g \in G \setminus N$ has the property that its conjugacy class is a union of cosets of $N$. Camina pairs have been generalized further to Camina triples. A Camina triple is a triple $(G, N, M)$ where $M \leq N$ are nontrivial normal proper subgroups of $G$ so that every element $g \in G \setminus N$ satisfies the property that its conjugacy class is a union of cosets of $M$.

In this paper the authors study groups $G$ with two nontrivial proper normal subgroups $M$ and $N$, with $M \leq N$, such that every element of $G \setminus N$ acts fixed-point-freely on $M$, such a triple $(G, N, M)$ is called a Frobenius triple. In [Math. Nachr. 18, 274–280 (1958; Zbl 0082.02501)], H. Wielandt studies finite groups $G$ possessing a subgroup $H$ for which every conjugate of $H$ intersects $H$ within some fixed normal subgroup $L$ of $H$. These triples $(G, H, L)$ have come to be known as Frobenius-Wielandt triples.

The main results proven in the article under review are. Theorem A: Let $(G, N, M)$ be a Camina triple. The following are equivalent: (i) $(G, N, M)$ is a Frobenius triple; (ii) $([G : N], [M]) = 1$; (iii) There exists a subgroup $H \leq G$ so that $G = HN$ and $H \cap M = 1$. Theorem B: Let $M$ and $N$ be nontrivial, proper normal subgroups of $G$ satisfying $M \leq N$. Let $H \leq G$ such that $G = HM$ and $H \cap M = 1$. Then $(G, N, M)$ is a Frobenius triple if and only if $(G, H, H \cap N)$ is Frobenius-Wielandt triple.

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MSC:

20C15 Ordinary representations and characters
20E34 General structure theorems for groups

Keywords:

Frobenius groups; Frobenius-Wielandt groups; Camina triples

Full Text: DOI

References:


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