
Summary: Quantum permutation matrices and quantum magic squares are generalizations of permutation matrices and magic squares, where the entries are no longer numbers but elements from arbitrary (non-commutative) algebras. The famous Birkhoff-von Neumann theorem characterizes magic squares as convex combinations of permutation matrices. In the non-commutative case, the corresponding question is as follows: Does every quantum magic square belong to the matrix convex hull of quantum permutation matrices? That is, does every quantum magic square dilate to a quantum permutation matrix? Here, we show that this is false even in the simplest non-commutative case. We also classify the quantum magic squares that dilate to a quantum permutation matrix with commuting entries and prove a quantitative lower bound on the diameter of this set. Finally, we conclude that not all Arveson extreme points of the free spectrahedron of quantum magic squares are quantum permutation matrices.

MSC:
05B15 Orthogonal arrays, Latin squares, Room squares
05B20 Combinatorial aspects of matrices (incidence, Hadamard, etc.)
15B51 Stochastic matrices
81P45 Quantum information, communication, networks (quantum-theoretic aspects)
81P68 Quantum computation

Keywords:
quantum permutation matrix; Birkhoff-von Neumann theorem; spectrahedron of quantum magic squares

Full Text: DOI

References:


[18] Paulsen, V.; Rahaman, M., Bisynchronous games and factorizable maps (2019)


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