Cartier, Pierre; Voros, André
Une nouvelle interprétation de la formule des traces de Selberg. (A new version of the Selberg trace formula). (French) [Zbl 0729.11024]

[For the entire collection see Zbl 0717.00009.]
The motivation of the paper under review is the observation that the Poisson summation formula is self-dual whereas the Selberg trace formula exhibits an asymmetry between the eigenvalue and length spectra. The aim is to better understand the analogies between the Selberg trace formula and the Poisson summation formula. The authors introduce new versions of the Selberg trace formula which add new flexibility in that they are based on a different class of test functions.

The first section is devoted to a study of the Poisson summation formula from the point of view of microlocal analysis. Section 2 contains some equivalent formulations of the Selberg trace formula, and the Selberg zeta function is discussed. Section 3 contains a careful discussion of (renormalized) characteristic determinants of operators acting on infinite-dimensional spaces. In particular, the Selberg zeta function is related with the determinant of the Laplacian on the corresponding compact Riemann surface, and the logarithmic derivative of the Gamma function in this formula is expressed by means of the determinant of the square root of \(-\Delta_{S^2} + 1/4\), where \(\Delta_{S^2}\) is the Laplacian on \(S^2\).

Section 4 contains the aforementioned new version of the Selberg trace formula. Contrary to the usual version, the test function \(h\) is now assumed to be holomorphic in a region containing \(|\arg r| < \pi/2 + \epsilon\) and containing the imaginary axis, and the main new trace formula reads (in standard notation \(\lambda_n = 1/4 + r_n^2\)):

\[
\sum_{n=0}^{\infty} h(r_n) = (2g - 2) \int_0^{\infty} r \tanh \pi r (r) dr + \int_0^{\infty} \frac{1}{2\pi i} (h(-it) - h(it)) d\log Z(1/2 + t),
\]

where \(Z\) is the Selberg zeta function and \(g\) the genus of the Riemann surface. (The precise definition of the latter integral is given by the authors.) The point is that the last integral gives another method of summation for the contribution of the closed geodesics.

Sections 5 and 6 contain applications of the new trace formula e.g. to the function \(\theta(t) = \sum_{n=0}^{\infty} \exp(-r_n t)\). This yields an interesting interpretation of the singularities of \(\theta\).

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MSC:
11F72 Spectral theory; trace formulas (e.g., that of Selberg)
11M41 Other Dirichlet series and zeta functions
58J50 Spectral problems; spectral geometry; scattering theory on manifolds
58J52 Determinants and determinant bundles, analytic torsion

Keywords:
Poisson summation formula; Selberg trace formula; microlocal analysis; Selberg zeta function; closed geodesics