Ghys, Etienne; de la Harpe, Pierre

Sur les groupes hyperboliques d'après Mikhael Gromov. (On the hyperbolic groups à la M. Gromov). (French) Zbl 0731.20025

Progress in Mathematics, 83. Boston, MA: Birkhäuser. vii, 285 p. sFr. 68.00 (1990).

[The articles of this volume will not be indexed individually.]

This book consists of a number of lectures by various authors who took part in a sustained seminar on the work of M. Gromov on hyperbolic groups, semihyperbolic groups and other related topics.

The basic object of study consists of a finitely generated group, Γ, together with a given finite set of generators, S. The "word length" metric makes Γ into a metric space. Two metric spaces, X and Y, are quasi-isomorphic if there exist maps $f : X \to Y$ and $g : Y \to X$ together with $\lambda > 0$, $C \geq 0$ such that

$$d(f(x_1), f(x_2)) \leq \lambda d(x_1, x_2) + C, \quad d(g(y_1), g(y_2)) \leq \lambda d(y_1, y_2) + C,$$

$$d(fg(y_1), y_1) \leq C \text{ and } d(gf(x_1), x_1) \leq C \text{ for all } x_1, x_2 \in X, \quad y_1, y_2 \in Y.$$

The quasi-isomorphism class of Γ is independent of the choice for S. Studying Γ (and its metric) via the associated Cayley graph, Gromov's work shows how the geometric notion of quasi- isomorphism succeeds in capturing many algebraic properties of Γ. For example:

Theorem: (i) If $[\Gamma : \Gamma_1] < \infty$ then Γ and Γ_1 are quasi-isomorphic, (ii) Let Γ_1, Γ_2 be quasi-isomorphic and torsion free. If Γ_1 is a nontrivial free product then so is Γ_2.

An example of an application of the method is the following celebrated result: Theorem: Γ has polynomial word growth if and only if it contains a nilpotent subgroup of finite index.

A metric space, X, with geodesics is called hyperbolic if any geodesic triangle in X satisfies the Rips condition with a positive constant. The group, Γ, is hyperbolic if its Cayley graph is hyperbolic. As Gromov's results are developed it becomes clear that the class of hyperbolic groups behaves much like the class of fundamental groups of negatively curved compact Riemannian manifolds.

Within its dozen chapters this volume covers many more properties and constructions than I can mention here; for example, every hyperbolic group has a unique "boundary" (which is the analogue of the boundary of the universal covering of a hyperbolic manifold). The careful treatment makes this book ideal for the newcomer to Gromov's theory.

In addition, the book concludes with a series of interesting appendices containing, inter alia, a number of new examples of hyperbolic groups.

Reviewer: V.P.Snaith (Hamilton/Ontario)

MSC:

20F38	Other groups related to topology or analysis
20F05	Generators, relations, and presentations of groups
20-02	Research exposition (monographs, survey articles) pertaining to group theory
53C21	Methods of global Riemannian geometry, including PDE methods; curvature restrictions
20E07	Subgroup theorems; subgroup growth
20E06	Free products of groups, free products with amalgamation, Higman-Neumann-Neumann extensions, and generalizations

Keywords:

Groupes hyperboliques; hyperbolic groups; semihyperbolic groups; finitely generated group; generators; Cayley graph; quasi-isomorphism; free product; polynomial word growth; nilpotent subgroup of finite index; fundamental groups; negatively curved compact Riemannian manifolds