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The authors consider the following equation

\[ D_i[a_{ij}(x,u)D_ju + b_i(x)u] - c(x,u) = f(x), \quad x \in \Omega \subset \mathbb{R}^m, \]

where \( \beta^{-1} a_{ij}(x,0)\xi_i \xi_j \leq a_{ij}(x,z)\xi_i \xi_j \leq \beta a_{ij}(x,0)\xi_i \xi_j, \quad \beta = \text{const} > 0, \lambda(x)\vert \xi \vert^2 \leq a_{ij}(x,0)\xi_i \xi_j, \lambda(x) \geq 0 \)
on \( \Omega \), with boundary conditions \( u(x) = 0, \quad x \in \Sigma_2 \cup \Sigma_3 \), where \( \Sigma_3 = \{ x \in \partial\Omega \mid a_{ij}(x,0)n_in_j > 0 \} \), \( \overrightarrow{n} = (n_1, \ldots, n_m) \) is the unit outward normal vector at \( x \in \partial\Omega \),

\( \Sigma_2 = \{ x \in \partial\Omega \setminus \Sigma_3 \mid b_i(x)n_i > 0 \} \), \( \Sigma_1 = \partial\Omega \setminus (\Sigma_2 \cup \Sigma_3) \).

Under some assumptions, the authors prove that the above problem has a weak solution (in some integral sense) which is also unique under additional restrictions. In this connection they establish an acute angle principle for weakly continuous mappings, discuss maximum and comparison principles and a maximum modulus estimate.

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MSC:

\begin{align*}
35J70 & \quad \text{Degenerate elliptic equations} \\
35B05 & \quad \text{Oscillation, zeros of solutions, mean value theorems, etc. in context of PDEs} \\
35D05 & \quad \text{Existence of generalized solutions of PDE (MSC2000)}
\end{align*}

Keywords:

Keldysh-Fichera boundary value problems; uniqueness; acute angle principle; maximum modulus estimate