On dispersable book embeddings. (English) Zbl 07318693


Summary: In a dispersable book embedding, the vertices of a given graph $G$ must be ordered along a line $\ell$, called spine, and the edges of $G$ must be drawn in different half-planes bounded by $\ell$, called pages of the book, such that: (i) no two edges of the same page cross, and (ii) the graph induced by the edges of each page is 1-regular (or equivalently, a matching). The minimum number of pages needed by any dispersable book embedding of $G$ is referred to as the dispersable book thickness $\text{dbt}(G)$ of $G$. Graph $G$ is called dispersable if $\text{dbt}(G) = \Delta(G)$ holds (note that $\Delta(G) \leq \text{dbt}(G)$ always holds).

Back in 1979, Bernhart and Kainen conjectured that any $\Delta$-regular bipartite graph $G$ is dispersable, i.e., $\text{dbt}(G) = \Delta$. In this paper, we employ a counting argument to disprove this conjecture for any fixed value of $\Delta \geq 3$. Additionally, for the cases $\Delta = 3$ and $\Delta = 4$ we present concrete counterexamples to the conjecture. In particular, we show that the Gray graph, which is 3-regular and bipartite, has dispersable book thickness four (with a computer-aided proof), while the Folkman graph, which is 4-regular and bipartite, has dispersable book thickness five (with a purely combinatorial proof). On the positive side, we prove that 3-regular bipartite planar graphs are dispersable.

MSC:
68Qxx Theory of computing

Keywords:
linear layouts; book embeddings; regular graphs

Software:
YalSAT; Treengeling; Plingeling; Lingeling; CaDiCaL

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